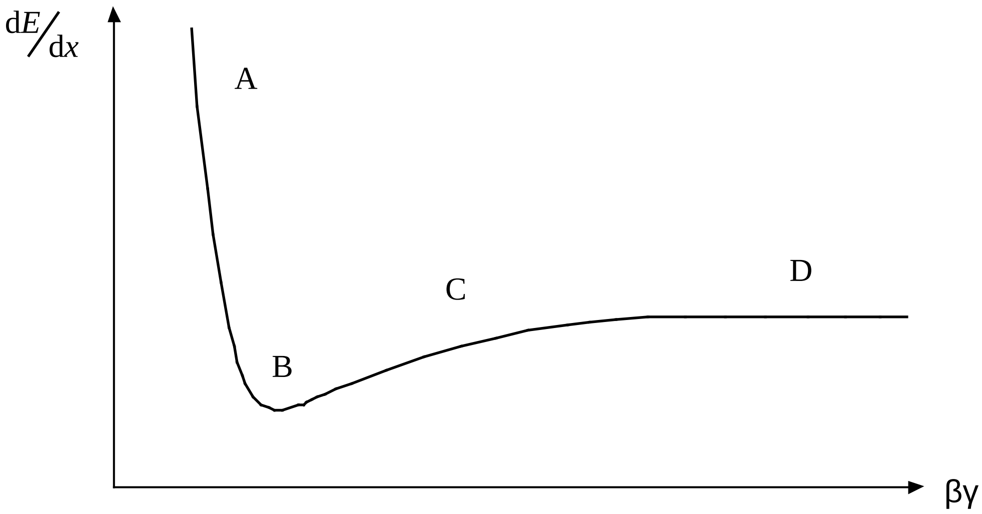
Specific Energy Loss as a function of Velocity

The Bethe-Bloch formula:

$$\overline{\Delta E} = 2C\frac{m_e c^2}{\beta^2}\frac{Zz^2}{A}\rho\Delta x\left[\ln\left(\frac{2\gamma^2\beta^2 m_e c^2}{I_0}\right) - \beta^2 - \frac{\varepsilon}{2} - \frac{\delta(\beta)}{2}\right]$$

gives the schematic result



A – rapid decrease $\frac{1}{\beta^2}$ $\left(=\frac{1}{v^2}\right)$ at non-relativistic velocities.

B – minimum at $E \approx 3\,Mc^2$ i.e. $\gamma \approx 3$.

$$\left(\frac{\mathrm{d}E}{\mathrm{d}x}\right)_{\min} \approx 1 - 1.5\ \text{MeV}\ (\text{g cm}^{-2})^{-1}$$ for most materials.

C – slow logarithmic "relativistic rise" $\ln(\gamma)$

for $\gamma \approx 1000$ $1.5 \times \left(\frac{\mathrm{d}E}{\mathrm{d}x}\right)_{\min}$ in very low density materials.

D – plateau as ionisation is limited by the *density effect*.

In solids, limited to only $\sim 1.1\left(\frac{\mathrm{d}E}{\mathrm{d}x}\right)_{\min}$ (reached at γ as low as 10).

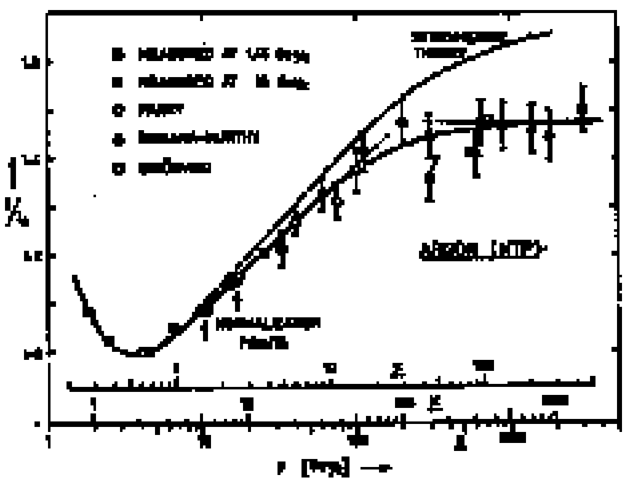


Fig. 1 Relativistic rise of the energy loss in argon, as a function of particle mass and momentum; the vertical scale gives the relative increase above the minimum of ionisation

N.B. The term minimum ionising particle (or mip) is often used for any particle with $\beta\gamma \geq 3$, as the increase above minimum is quite small, compared with the very large changes for non-relativistic particles below the minimum.

Where Does the Energy Go?

Most of the energy lost by the incident particle is used in creating ion pairs. A smaller amount, from the large impact parameter collisions, causes atomic excitation where the electron does not receive enough energy to leave the atom.

From equation (3), we saw that

$$P(E_\mathrm{e})\,\mathrm{d}E \propto \frac{\mathrm{d}E}{E_\mathrm{e}^2}$$

This is a steeply falling spectrum – i.e. most electrons have low energy.

Higher energy electrons are known as *delta-rays*. These are also ionising, and can lead to a 3 to 4-fold increase in the number of ion-pairs over that deposited in the primary ionisation. However, the number of ion pairs is still proportional to the energy lost by the primary particle.

The Range of Slow Particles

We have seen that slower particles have a high rate of energy loss. If a particle has a small enough initial kinetic energy, it may be stopped completely by a thick layer of material. The range R of the particle is then given by

$$R = \int_T^0 -\left(\frac{\mathrm{d}E}{\mathrm{d}x}\right)^{-1} \mathrm{d}E$$

It can easily be shown that this may be expressed as

$$R = \frac{M}{z^2} \int_{v_0}^0 f(v)\,\mathrm{d}v$$

where the integral is a universal function, independent of the properties of the particle.

Thus, for a given initial velocity,

$$R \propto \frac{M}{z^2}.$$

The range can therefore be used to identify the type of particle, provided its initial velocity is known.

<u>Fluctuations in Energy Loss</u> – *not examinable!!*

So far, we have only considered the average energy loss experienced by a particle in passing through a layer of material of a given thickness. However, the energy loss process is statistical. Large fluctuations are mainly due to a small number of energetic knock-on electrons known as δ-rays.

Consider a "thin" absorber, such that $\overline{\Delta E} \ll E$

1) If $\overline{\Delta E} \gg E_{\max}$, then many collisions are involved, and the distribution is Gaussian

then $\left\langle (\Delta E)^2 \right\rangle = \int_0^{E_{\max}} E^2 P(E)\,\mathrm{d}E$ (note that this integral does not blow up at the lower limit)

hence $(\text{width})^2$ i.e. $\left\langle (\Delta E)^2 \right\rangle - \left\langle \Delta E \right\rangle^2$ is $\dfrac{C m_e c^2 Z z^2 E_{\max}}{\beta^2 A}\left(1 - \dfrac{\beta^2}{2}\right)\rho\Delta x$ (see Rossi p. 31)

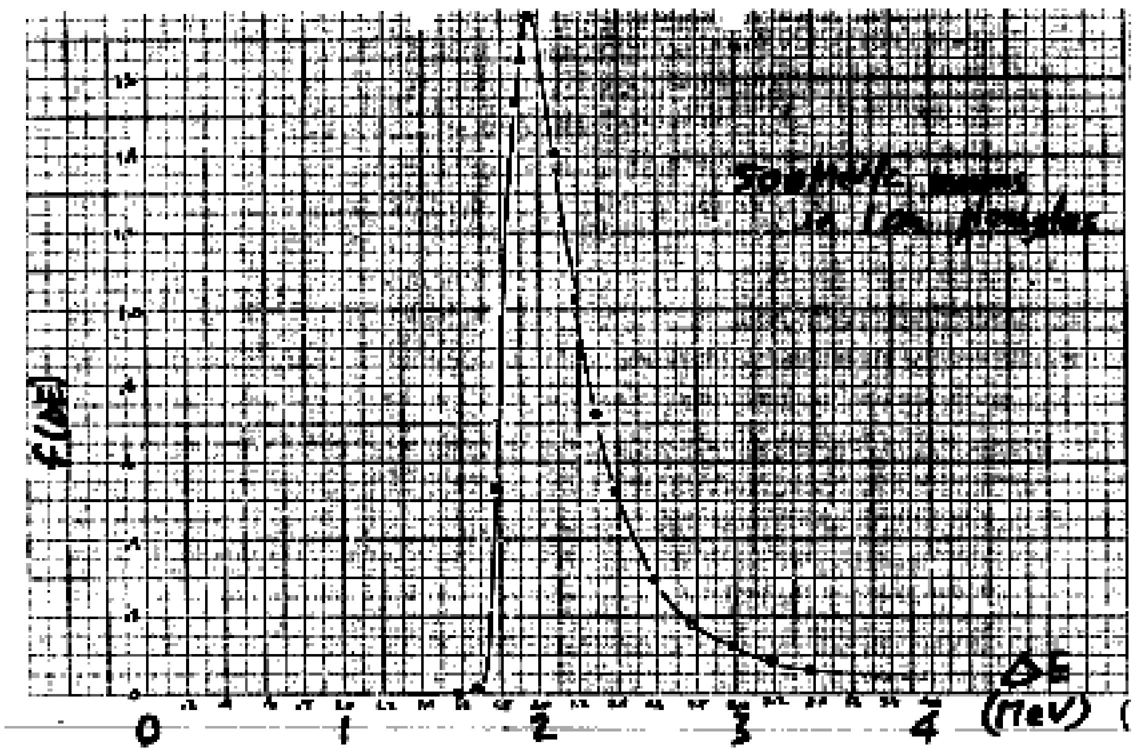
and for a mip this is $\sim \dfrac{C m_e c^2 Z z^2 E_{\max}\rho\Delta x}{2A}$.

[Note: For a 10 GeV proton, $E_{\max}$ is about 100 MeV. $\overline{\Delta E} \gg E_{\max}$ therefore requires metres of a material like plastic scintillator!]

2) If $\dfrac{\overline{\Delta E}}{\left[\ln\left(\dfrac{2\gamma^2\beta^2 m_e c^2}{I_0}\right) - \beta^2 - \dfrac{\varepsilon}{2} - \dfrac{\delta(\beta)}{2}\right]} \le 0.01 E_{\max}$, then a distribution known as the <u>Landau distribution</u> can be used.

E.g. for a mip (e.g. 3 GeV proton) and 1 cm of plastic scintillator, $\overline{\Delta E}$ ~ 1.4 MeV, $\dfrac{\overline{\Delta E}}{[\;\;]}$ ~ 0.15 MeV, $E_{\max}$ is then about 9 MeV, and the above condition is (just about!) ok.

See the plot of the Landau distribution.



Note the very long tail to the Landau distribution, of low-probability events with very large energy losses. However, the energy deposited in a thin layer is not the same as that lost by the traversing particle, as energetic δ-rays (responsible for much of the large energy-loss tail) can escape from the layer. The fluctuations in energy deposited are therefore significantly less than those in energy lost.

[The Landau distribution can be parametrised as $f(\lambda) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\lambda + e^{-\lambda}\right)}$

where $\lambda = \left(\Delta E - \Delta E_{\text{m.p.}}\right) \Big/ C \frac{m_e c^2}{\beta^2} \frac{Zz^2}{A} \rho \Delta x$ and $\Delta E_{\text{m.p.}}$ is the most probable energy loss.]

3) A more general energy-loss distribution is given by the Vavilov distribution, which can be found in specialised text books.

However, both the Landau and Vavilov distributions employ approximations which break down for *very* thin layers, with small $\overline{\Delta E}$, as atomic excitation effects are not dealt with properly.