Interaction of a Electrons & Photons with Matter

a) Electrons

Electrons are charged particles, and so interact with matter through their Coulomb field in the way that we have already discussed for other charged particles. However, their very low mass leads to a number of modifications to their rate of energy loss through ionisation:

- they are "always" relativistic
- they can lose almost all their energy in a single interaction with an atomic electron (unlike heavier particles – see earlier notes)
- quantum mechanical treatment of the scattering has to allow for the interaction of two identical particles.

As a result, Bethe theory leads to a modification of the formula for the energy loss of a charged particle for the case of an electron:

$$\left.\frac{\mathrm{d}E}{\mathrm{d}x}\right)_{\text{ion}} = -2Cm_e c^2 \frac{Z}{A}\rho\left[\ln\left(\frac{\pi\gamma^{3/2} m_e c^2}{I_0}\right) - \frac{a}{2} - \frac{\varepsilon}{2} - \frac{\delta}{2}\right]$$

Here β has been set equal to 1, and a has the value 2.9 for e^- and 3.6 for e^+. All other symbols have the same definition as in the previous derivation of energy loss, including $C = 2\pi N_A \left(\frac{e^2}{4\pi\varepsilon_0 m_e c^2}\right)^2 = 0.3006 \left(\text{g cm}^{-2}\right)^{-1}$. Note that here, for electrons, the power of γ in the logarithm is $3/2$ compared with 2 for heavy particles. As a comparison, for heavy particles with $\beta = 1$ and $z = 1$ we previously had

$$\left.\frac{\mathrm{d}E}{\mathrm{d}x}\right)_{\text{ion}} = -2Cm_e c^2 \frac{Z}{A}\rho\left[\ln\left(\frac{2\gamma^2 m_e c^2}{I_0}\right) - 1 - \frac{\varepsilon}{2} - \frac{\delta}{2}\right]$$

There is, however, a much bigger effect when electrons interact with matter. Because of their low mass, they can undergo very large accelerations in the field of nuclei, leading to bremsstrahlung (or "braking radiation"). Passage through matter therefore results in the emission of high energy photons or γ-rays with a spectrum $\frac{\mathrm{d}N_\gamma}{\mathrm{d}E_\gamma}$ proportional to $\frac{1}{E_\gamma}$. As in the case of ionisation loss, the importance of screening of the nucleus by atomic electrons depends on γ, and two regimes can be identified.

i) for $1 << \gamma << \frac{137}{Z^{1/3}}$ (i.e. E << 70 MeV in H, E << 16 MeV in Pb)

$$\left.\frac{\mathrm{d}E}{\mathrm{d}x}\right)_{\text{rad}} = -\frac{4}{137}\frac{N_A}{A} Z^2 \rho \left(\frac{e^2}{4\pi\varepsilon_0 m_e c^2}\right)^2 E\left[\ln(2\gamma) - \frac{1}{3}\right]$$

$$= -\frac{2}{\pi}\frac{C}{137}\frac{Z^2}{A}\rho E\left[\ln(2\gamma) - \frac{1}{3}\right]$$

ii) for $\gamma >> \frac{137}{Z^{1/3}}$ (i.e. the normal high energy case)

$$\left.\frac{\mathrm{d}E}{\mathrm{d}x}\right)_{\text{rad}} = -\frac{2}{\pi}\frac{C}{137}\frac{Z^2}{A}\rho\left(E - m_e c^2\right)\left[\ln\left(\frac{183}{Z^{1/3}}\right) + \frac{1}{18}\right]$$

When atomic electrons are considered in addition to the nuclear charge, the above expressions must be modified by replacing Z^2 with $Z(Z+1)$.

In the second case, a little manipulation and using $E >> m_e c^2$ leads to the expression

$$\left.\frac{\mathrm{d}E}{\rho \mathrm{d}x}\right)_{\mathrm{rad}} = -\frac{E}{X_0}(1+b)$$

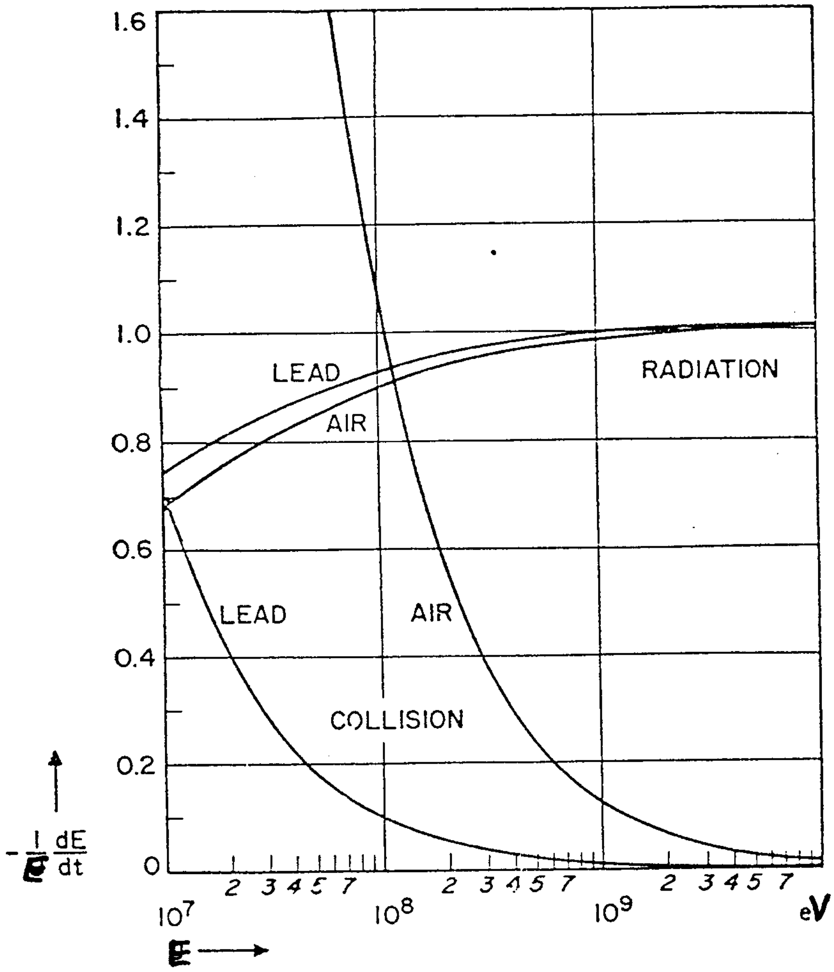
where $$b = \frac{1}{18\ln\left(183/Z^{1/3}\right)} \approx 0.01 \text{ which is negligible,}$$

the radiation length $$X_0 = \frac{137A}{4Z(Z+1)}\left(\frac{4\pi\varepsilon_0 m_e c^2}{e^2}\right)^2 \frac{1}{N_A \ln(183/Z^{1/3})}$$

Hence the mean energy of an electron, of initial energy E_0, after losing energy by radiation through a distance Δx is given by

$$\overline{E} \approx E_0 \exp\left(-\frac{\rho \Delta x}{X_0}\right)$$

Fractional energy loss
per radiation length



Fractional energy loss per radiation length for electrons passing through air and lead, shown for each of radiation and ionisation (collision) loss mechanisms.

Note that for high energy electrons $\left.\frac{\mathrm{d}E}{\mathrm{d}x}\right)_{\mathrm{rad}} >> \left.\frac{\mathrm{d}E}{\mathrm{d}x}\right)_{\mathrm{ion}}$. Energy loss by ionisation can therefore be ignored until the electron's energy drops to a critical energy, $E_c \approx \frac{600 \text{ MeV}}{Z}$.

Below this value, energy loss by ionisation dominates.

b) Photons

There are three possible processes which can occur when photons interact with matter.

i) Photoelectric Effect – The energy of the photon supplies the binding energy of an electron in an atom.

$$\gamma \;+\; e \;\rightarrow\; e_{\text{free}}$$

(The cross section σ for this process is proportional to E_γ^{-3} and to Z^4.)

ii) Compton Scattering – elastic scattering of a photon from an electron

$$\gamma \;+\; e \;\rightarrow\; \gamma \;+\; e \quad \text{when} \quad E_\gamma >> I_0$$

(The cross section is approximately proportional to E_γ^{-1} – see appendix.)

iii) Pair Production – $\gamma \rightarrow e^+ + e^-$ occurs when $E_\gamma > 2m_e$ and is the dominant interaction for photons of energy above about 10 MeV.

Note that it is not possible to conserve both energy and momentum if this process occurs in free space – it can only happen in the strong electric field around a nucleus, where the recoiling nucleus can absorb significant momentum but negligible energy.

Note also that the diagram for pair production is very similar to that for electron bremsstrahlung, and calculations of the probability of pair production as a gamma ray traverses matter result in the following similar expressions.

i) In the low energy case of $1 << \dfrac{E}{m_e c^2} << \dfrac{137}{Z^{1/3}}$

$$\text{Probability of photon conversion} = \frac{2}{\pi}\frac{C}{137}\frac{Z^2}{A}\rho\Delta x\left[\frac{7}{9}\ln\left(\frac{2E}{m_e c^2}\right) - \frac{109}{54}\right]$$

ii) for $\dfrac{E}{m_e c^2} >> \dfrac{137}{Z^{1/3}}$ (the usual high energy case)

$$\text{Probability of photon conversion} = \frac{2}{\pi}\frac{C}{137}\frac{Z^2}{A}\rho\Delta x\left[\frac{7}{9}\ln\left(\frac{183}{Z^{1/3}}\right) - \frac{1}{54}\right]$$

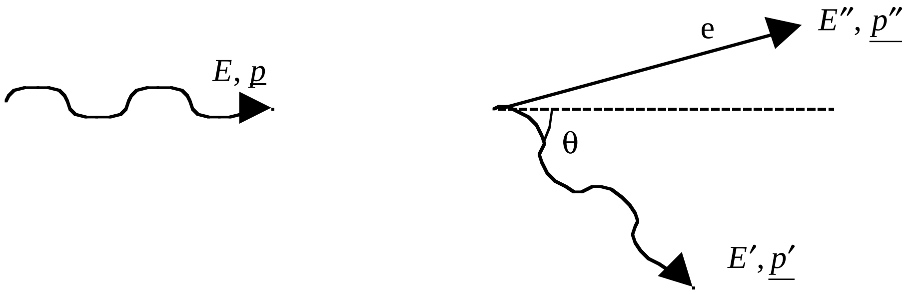
Hence $$\frac{dN_\gamma}{\rho dx} = -\frac{N_\gamma}{X_0}\left(\frac{7}{9} - \frac{b}{3}\right),$$ with b as before ~0.01.

Therefore, for a beam of photons, $$\overline{N_\gamma} \approx N_0 \exp\left(-\frac{7}{9}\frac{\rho\Delta x}{X_0}\right)$$

$\dfrac{9}{7}X_0$ is known as the conversion length.

If the energy flux carried by beams of electrons and photons is compared, we see that for electrons the $1/e$ decay length is X_0 as the *energy* of the electrons is reduced, while for photons it is $9/7\,X_0$ as the *number* of the photons is reduced.

Appendix – Compton Scattering



First let us consider the relationship between scattering angle and energy loss:

Conservation of energy: $E'' = E - E' - m_e c^2$

Conservation of momentum $\underline{p''} = \underline{p} - \underline{p'}$ $\quad p = \dfrac{E}{c}$ for photon

$\Rightarrow \quad p''^2 = p^2 + p'^2 - 2pp'\cos\theta \qquad p''^2 c^2 = E''^2 - m_e^2 c^4$ for electron

$\Rightarrow \quad E''^2 - m_e^2 c^4 = E^2 + E'^2 - 2EE'^2 \cos\theta$

$\Rightarrow \quad E^2 - E'^2 + m_e^2 c^4 - 2EE' + 2Em_e c^2 - 2E' m_e c^2 - m_e^2 c^4 = E^2 + E'^2 - 2EE'^2 \cos\theta$

$\Rightarrow \quad E' = \dfrac{Em_e c^2}{m_e c^2 + E(1-\cos\theta)}$

Therefore: $E'_{\max} = E$ when $\theta = 0$ (effectively no scattering)

$E'_{\min} = \dfrac{Em_e c^2}{m_e c^2 + 2E} \approx \dfrac{m_e c^2}{2}$ when $\theta = 180°$

The *probability* of a scattering from photon energy E to E' was calculated by Klein and Nishina. Here we simply present the result:

$$P(E \to E') = \pi N_A \frac{Z}{A}\left(\frac{e^2}{4\pi\varepsilon_0 m_e c^2}\right)^2 \frac{m_e c^2}{EE'}\left(1 + \left(\frac{E'}{E}\right)^2 - \frac{E'\sin^2\theta}{E}\right)\rho\Delta x$$

$$= \frac{C}{2}\frac{Z}{A}\frac{m_e c^2}{EE'}\left(1 + \left(\frac{E'}{E}\right)^2 - \frac{E'\sin^2\theta}{E}\right)\rho\Delta x$$

In this expression, we can neglect the term $\dfrac{E'\sin^2\theta}{E}$ as E' is small except when θ is small.

The *overall* probability of Compton scattering $= \displaystyle\int_{m_e c^2/2}^{E} P(E \to E')\,\mathrm{d}E'$

$$\approx \frac{C}{2}\frac{Z}{A}\frac{m_e c^2}{E}\left(\ln\left(\frac{2E}{m_e c^2}\right) + \frac{1}{2}\right)\rho\Delta x$$

i.e. since the logarithmic term is slowly varying, to first approximation

$$\sigma \propto \frac{1}{E_\gamma}.$$