Fundamental Interactions

2) Weak Interactions

Three properties of certain decays led to the postulation of the weak interaction;

a) Low rates or long lifetimes.
b) Violation of certain "conservation laws" – changes in parity, strangeness and isospin.
c) Frequent involvement of neutrinos, particles which are not involved in any other sort of interactions.

The original Fermi theory of beta decay involved a point like "contact interaction" with a coupling constant

$$G = \frac{10^{-5}}{m_{\mathrm{p}}^2} \qquad \text{(i.e. weak compared with } {}^{1}\!/_{137}\text{).}$$

This value of G is compatible with all the observed decay rates for the above processes. However, if we calculate neutrino electron scattering, ν e $\rightarrow$ ν e, it is found that the cross-section rises with neutrino energy E as

$$\sigma_{\mathrm{tot}} = \frac{2G^2 m_{\mathrm{e}} E}{\pi} \qquad \text{tending to infinity with } E!$$

Very general scattering theory provides a limit on the maximum possible elastic cross-section, purely based on the conservation of probability, or "unitarity", and this is violated when $m_{\mathrm{e}} E > {}^{\pi}\!/_{G}$. A solution to this problem, as we have already suggested, is to replace the 4-fermion contact interaction with boson exchange, as in QED. The exchange of a massless boson would not be compatible with low energy behaviour (where the cross section does rise with E), and the exchanged "weak intermediate vector boson", W, must have a considerable mass. The propagator that this introduces is $\frac{1}{q^2 + m_{\mathrm{W}}^2}$, and this acts as a constant at low energy (or low q^2), but falls as $1/q^2$ at very high q^2.

The effective strength of the weak interaction at low energies thus depends both on the coupling of the W to fermions, g, and upon its mass. At low energies, the cross section is proportional to

$$G^2 \sim \lim_{q^2 \to 0} \left(\frac{g^4}{\left(q^2 + m_{\mathrm{W}}^2\right)^2} \right) = \frac{g^4}{m_{\mathrm{W}}^4}. \qquad \text{i.e. } G \sim \frac{g^2}{m_{\mathrm{W}}^2},$$

so if m_{W} is large, the weak coupling g is not necessarily very small.

The weak interaction responsible for beta decay involves a charged (or charge-changing) current, and the W must exist as W^+ and W^-. This suggested there might also be a weak neutral current, propagated by a third boson, the Z^0. Evidence for this was first observed at CERN in 1973 in the form of neutrino interactions ν_μ N $\rightarrow$ ν_μ X (where N is a nucleus and X a hadronic system) and ν_μ e $\rightarrow$ ν_μ e. However, this exposed a new theoretical problem. At sufficiently high energy, it should be possible to produce real W^+ W^- pairs in e^+ e^- annihilation through the Z^0, and a calculation shows that at very high energies this cross section again becomes unphysically large. The diagram for this process would be cancelled by that corresponding to electromagnetic annihilation to W^+ W^- through a photon, but this cancellation will only occur if the weak coupling constant g is equal to the electromagnetic coupling, $\sqrt{\alpha}$. (Note that at such very high energies, the mass of the Z^0 becomes insignificant.)

In the 1960's, Glashow, Salam and Weinberg proposed the unification of the electromagnetic and weak interactions as a single gauge theory with a common coupling constant. This implied that the mass of the W and Z must be about 90 GeV/c^2. (At low energies, the W and Z cannot be produced as real particles, and the electromagnetic and weak interactions appear as separate processes.) The first real weak bosons were produced at the CERN antiproton-proton collider, which effectively allowed antiquarks and quarks to interact. In 1982, processes such as $\mathrm{d}\,\bar{\mathrm{u}} \rightarrow \mathrm{W}^- \rightarrow \mathrm{e}^-\,\bar{\nu}_\mathrm{e}$ were observed in the UA1 and UA2 experiments, while the next year the rarer but cleaner signature $\mathrm{q}\,\bar{\mathrm{q}} \rightarrow \mathrm{Z}^0 \rightarrow \mathrm{e}^+\,\mathrm{e}^-$ was seen. In the late 1980s, a large $\mathrm{e}^+\,\mathrm{e}^-$ collider, LEP, was built at CERN, allowing the “mass production” of Z^0 bosons. By colliding the particles with exactly the right energy to supply the Z mass, a large resonance in the cross section occurs, and now several million Z^0 decays have been observed, shared between the 4 large experiments.

The Z couples to all fermion-antifermion pairs, including neutral neutrinos. The decays to neutrinos are basically undetectable, as νs leave no tracks and can pass through large amounts of material without interacting, but nonetheless they modify the properties of the Z, and results on ν production were some of the most important to emerge in the early days of LEP! The Z is a very short-lived particle, with a lifetime of only 2.6×10^{-25} s. The uncertainty principle implies a relationship between the uncertainty in energy (and hence mass) and that in time. Thus, in $\mathrm{e}^+\,\mathrm{e}^-$ annihilation, the Z^0 is seen with a width in mass of about 2.5 GeV (FWHM), as shown in fig. 7. Decays of the Z into neutrinos have two effects. Each species of neutrino decreases the number of *visible* Z decays by 13% and also shortens the lifetime, and hence increases the width by 0.176 GeV. Measurements by the ALEPH collaboration at LEP show that the data imply $N_\nu = 2.994 \pm 0.012$ i.e. 3! There are therefore no new generations of lepton and neutrino (unless the mass of the new neutrino is greater than 45 GeV/c^2 – very different from the three known species, which are almost massless).

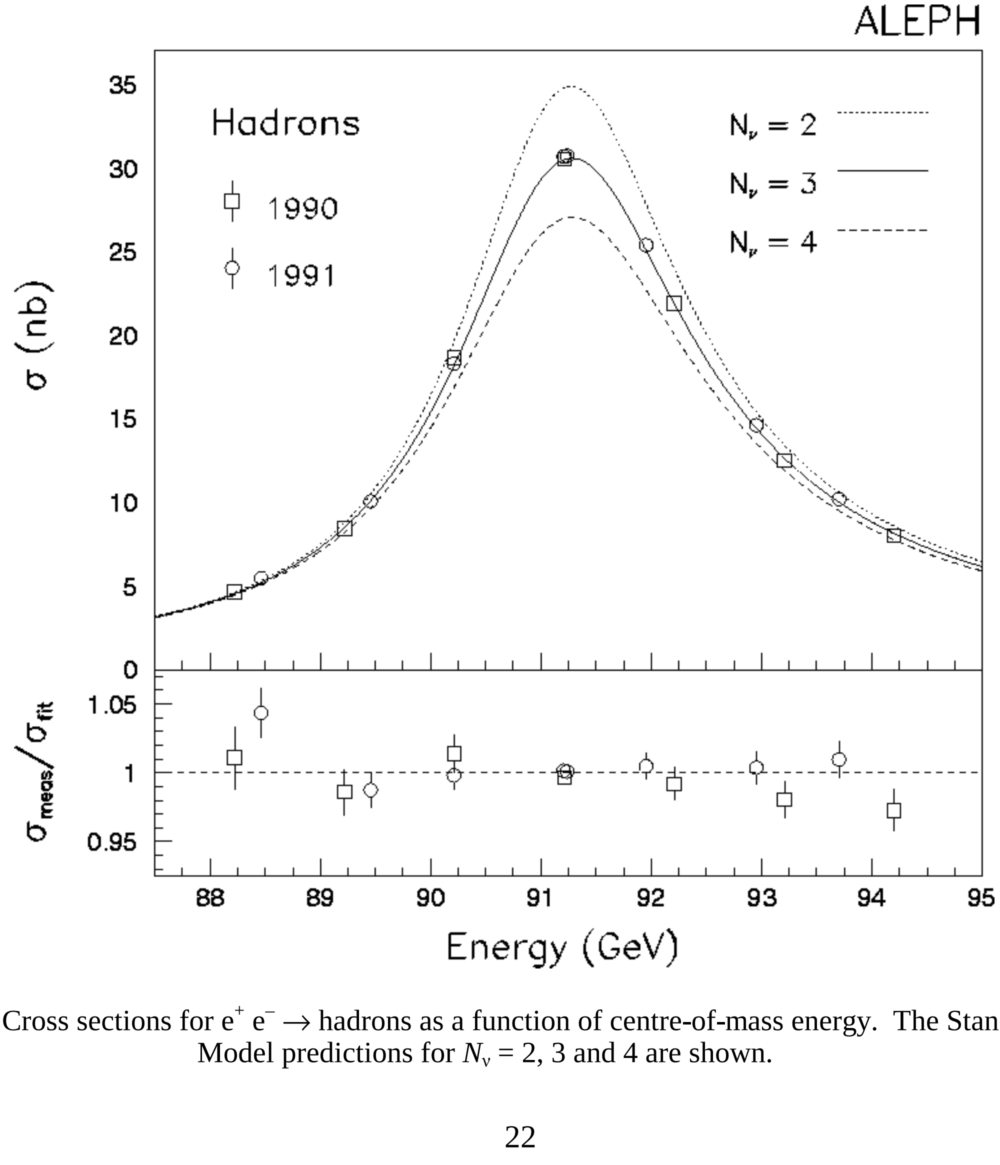

Fig. 7 Cross sections for $\mathrm{e}^+\,\mathrm{e}^- \rightarrow$ hadrons as a function of centre-of-mass energy. The Standard Model predictions for $N_\nu = 2$, 3 and 4 are shown.