Fundamental Interactions

1) Quantum ElectroDynamics (QED)

In the model of interactions we have proposed, a charge, for example, interacts by emitting and absorbing virtual photons. We now examine the possibility of observing consequences of this which are not predicted by standard quantum mechanics. Our model of an electron interacting with an external electromagnetic field involves it in absorbing a virtual photon, and thus changing its momentum. However, other *internal* interactions can occur. An electron, of momentum p, may emit a virtual photon of momentum k, and hence continue with a reduced momentum $p–k$ until it reabsorbs the virtual photon. Similarly a photon of momentum k may convert into a virtual electron positron pair, with the electron and positron sharing the original momentum, until the virtual pair recombine and produce the photon again. Indeed, more complicated cases may occur, involving combinations of emission of virtual photons with virtual pair productions. Each coupling of a photon to a fermion line, known as a vertex, involves a factor $\sqrt{\alpha}$ in the amplitude, so α in the cross-section, where

$$\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c} = \frac{1}{137}$$

The above internal interactions are known as self-energy terms. The first contributes to the apparent mass of the electron, while the second contributes to the effective charge (and is responsible for the process known as “vacuum polarisation”). We can sum over all diagrams, and integrate over all internal loop momenta, in an attempt to calculate the effective mass and charge. However, naive attempts to do this result in values that are infinitely large! A little thought shows that it is not reasonable to put in the measured values of m and e as the *bare* parameters of the theory, and then *calculate* some modified, effective values. Whenever we do an experiment with an electron, it is surrounded by its cloud of virtual photons and pairs – it is $m_{\text{effective}}$ that we measure in experiments! A rigorous mathematical process known as renormalisation allows us to use the physical mass and charge and (for most processes) ignore *internal* loops in the particle lines. *External* loops (e.g. those coupling incoming and outgoing particles) will, however, be expected to have a finite, observable effect.

There are several properties which exhibit the quantum nature of the electromagnetic interaction. One of these is the magnetic moment of charged leptons, in particular the muon. The Dirac equation for a point-like electron or muon, which arises from a relativistic quantum mechanical treatment of the particle but without the use of a field theoretical approach to describe interactions, predicts that a component of the lepton’s intrinsic magnetic moment must be $\mu_z = \pm \mu_B$, where μ_B is the Bohr magneton,

$$\mu_B = \frac{e\hbar}{2m}$$

or, by analogy with atomic physics, writing $\underline{\mu} = g\,\mu_B\,\underline{s}$, with $s_z = \pm\tfrac{1}{2}$

then $g = 2$.

Without QED, the prediction is therefore that $g = 2$. We can allow for the existence of extra physics by writing g as $2 \times (1 + a)$, where a is known as the anomalous magnetic moment. The full QED calculation is very involved, but field theoretical considerations lead us to the expectation that the highest order contribution to a will be of the order of α, $\tfrac{1}{137}$. We can even produce qualitative explanations of why these higher order processes might modify the observed magnetic moment. An electron is always surrounded by a cloud of virtual photons. The charge is carried by the electron, but the energy, and hence mass, is shared between electron and photons. The value of e/m for the “bare” electron is therefore slightly increased, and it is this which will be measured in the interaction with an external magnetic field. Alternatively, when a lepton, of spin $\tfrac{1}{2}$, emits a photon, of spin 1, its spin, and hence magnetic moment, must be flipped. The average magnetic moment of a real lepton might therefore be reduced, when compared with the expectation for a bare particle. Another effect gives a larger, positive contribution to *a*. A classical picture of the magnetic moment of a particle is that it arises from a current loop around the centre of mass of the particle. Emission and absorption of virtual photons leads to a jitter in the position of the centre of the loop, so increasing the effective area of the loop and increasing μ.

The full QED calculation for the leading order contribution to *a* is $0.5\alpha/\pi$.

Higher order terms depend on the lepton mass (e.g. see Perkins section 6.5),

$$a_e \equiv \left(\frac{g-2}{2}\right)_e^{\text{QED}} = 0.5\frac{\alpha}{\pi} - 0.32848\left(\frac{\alpha}{\pi}\right)^2 + 1.19\left(\frac{\alpha}{\pi}\right)^3 \cdots = (1159652.4 \pm 0.4)\times 10^{-9}$$

$$a_\mu \equiv \left(\frac{g-2}{2}\right)_\mu^{\text{QED}} = 0.5\frac{\alpha}{\pi} + 0.76578\left(\frac{\alpha}{\pi}\right)^2 + 24.45\left(\frac{\alpha}{\pi}\right)^3 \cdots = (1165851.7 \pm 2.3)\times 10^{-9}$$

Experiment

A pioneering experiment to measure *g*–2 for the muon was performed by a team including the late Prof. Combley of this department. A polarised beam of muons was circulated in a circular orbit in a uniform magnetic field. The interaction between the magnetic field and the magnetic moment exerts a torque on the muon, causing the spin direction to process at a rate which depends on the magnetic field (Larmor and Thomas precessions). If $g = 2$, the momentum and spin would turn at precisely the same rate, and so the polarisation would not change.

The actual polarisation of the muons was measured through their parity non-conserving weak decays, $\mu^+ \to e^+ \, \bar{\nu}_\mu \, \nu_e$. This process is mediated by a W boson (as will be discussed in the section of the course on weak interactions), and it can easily be shown, by considering the helicities of the neutrino and antineutrino, that, in the rest frame of the μ^+, the e^+ direction tends to follow the direction of the muon's spin. In the lab frame, the highest energy electrons have their decay momentum parallel to the muon's momentum, so they tag times when the muon's spin is parallel to its momentum. Exploiting these facts, the precession rate, and so the value of *a*, can be measured.

Experimentally $a = (1165924. \pm 9.)\times 10^{-9}$.
A QED calculation, including higher orders, predicts $a = (1165851.7 \pm 2.3)\times 10^{-9}$,
leaving a discrepancy of $(72.3 \pm 9.3)\times 10^{-9}$.

Though the above theoretical result is very close to the experimental value, there is still a significant difference – something must be missing! This is the effect of other (strongly interacting) particles, which can also contribute to vacuum polarisation but are not included in the QED calculation. A calculation of this additional term yields $(70.2 \pm 8.0)\times 10^{-9}$,
nicely accounting for the above discrepancy.

(The above results are taken from the CERN *g*–2 experiment, published in 1981. In 2001, a similar experiment at Brookhaven compared more precise experimental results with better theoretical calculations – incorporating higher order Feynman diagrams – and tantalisingly found a 3 standard deviation discrepancy. This could be due to the existence of undiscovered particles beyond the standard model of particle physics. A third-generation experiment at Fermilab has recently started taking data in an attempt to elucidate the situation.)

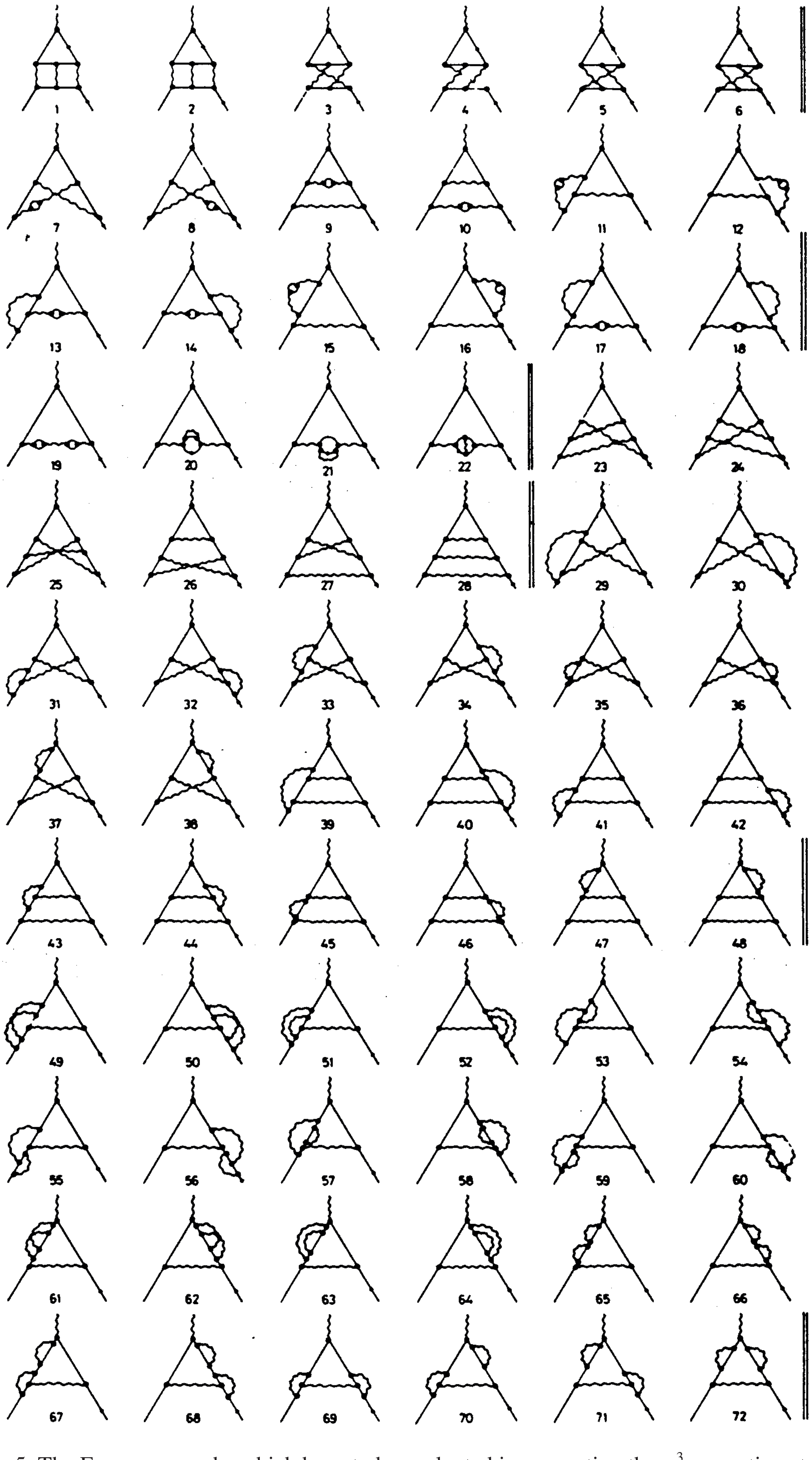

Fig. 5 The Feynman graphs which have to be evaluated in computing the α^3 corrections to the lepton magnetic moments. (From Perkins, ed. 3, p. 322.)

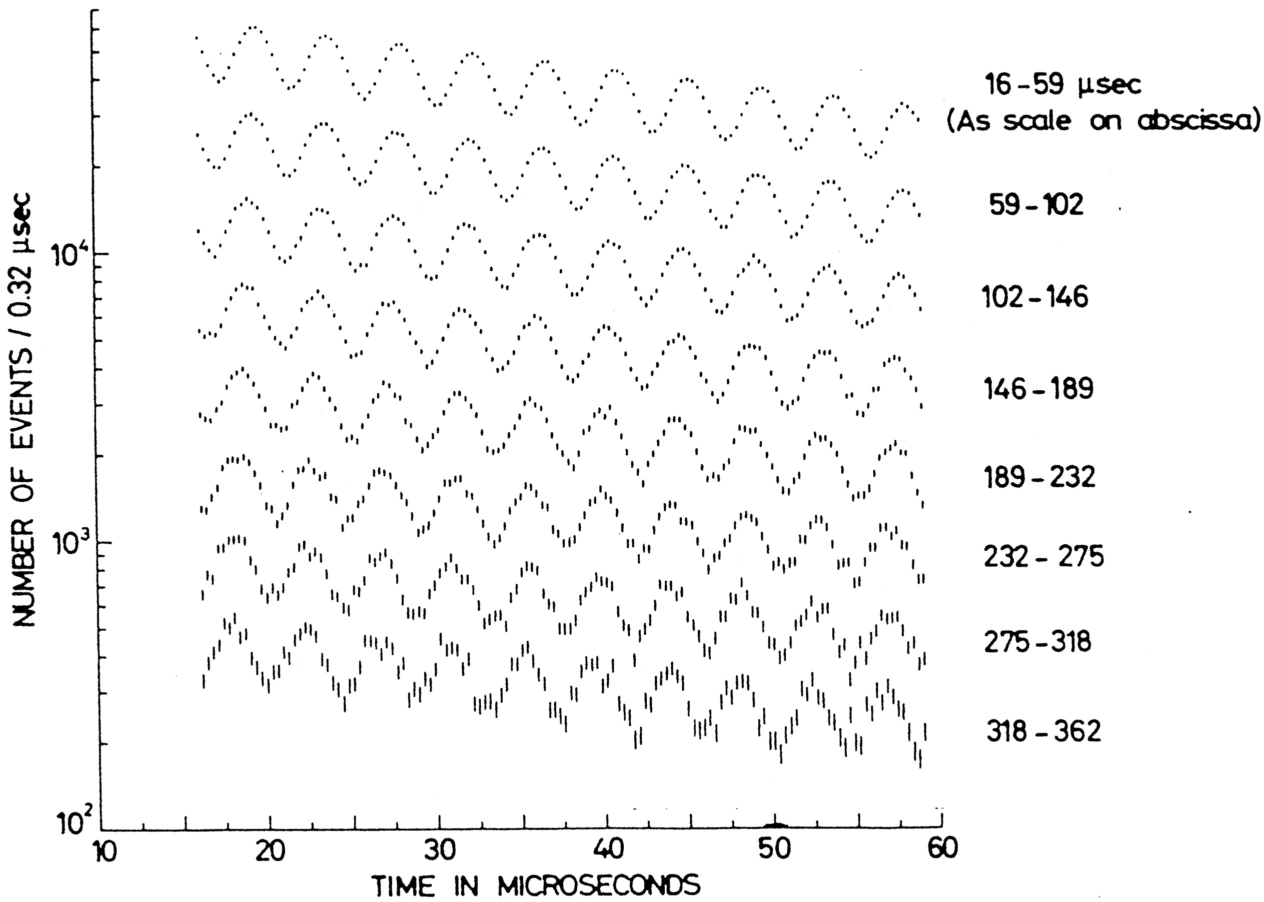

Fig. 6 Time dependence of the electron counting rate from the decay of muons in the CERN $g-2$ experiment. Note that the abscissa is folded every 43 µs, in order to display the full time range of 360 µs. The general exponential decrease corresponds to the loss of muons by radioactive decay, with a mean lifetime dilated by the relativistic γ-factor of 30. (Note that at the time this experiment provided the most precise (0.1% accuracy) check of Einstein's time-dilation formula.) The overall decrease is modulated by the $g-2$ frequency. (From F. Combley *et al.*, Physics Reports **68** (1981) 93-119.)