Properties of Quarks

In the earlier part of this course, we have discussed three families of leptons but principally concentrated on one doublet of quarks, the u and d. We will now introduce other types of quarks, along with the new quantum numbers which characterise them.

Isospin

It was noticed that many groupings of particles of similar mass and properties fitted in to common patterns. One way to characterise these is using isotopic spin or isospin, I. This quantity has nothing to do with the real spin of the particle, but obeys the same addition laws as the quantum mechanical rules for adding angular momentum or spin. When the orientation of an isospin vector is considered, it is in some hypothetical space, not in terms of the x, y and z axes of normal co-ordinates.

Nucleons (p, n), pi mesons (π^+, π^0, π^-) and the baryons known as Δ (Δ^{++}, Δ^+, Δ^0, Δ^-) are three examples of groups of similar mass particles differing in charge by one unit. The charge Q in each case can be considered as due to the orientation of an "isospin vector" in some hypothetical space, such that Q depends on the third component I_3. Thus the nucleons belong to an isospin doublet: $\mathrm{p} \equiv \left|I, I_3\right\rangle = \left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle$; $\mathrm{n} = \left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle$. Similarly the pions form an isospin triplet, $\pi^+ = \left|1,1\right\rangle$; $\pi^0 = \left|1,0\right\rangle$; $\pi^- = \left|1,-1\right\rangle$. The Δ forms a quadruplet with $I = \tfrac{3}{2}$. The rule for electric charge can then be written $Q = e\left(\tfrac{1}{2}B + I_3\right)$, where B is the baryon number which is 1 for nucleons and the Δ and 0 for mesons such as the π. In terms of quarks, the u and d form an isospin doublet, $\mathrm{u} = \left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle$; $\mathrm{d} = \left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle$ (both with $B = \tfrac{1}{3}$).

Three quarks with $I = \tfrac{1}{2}$ can combine to form $I_{\mathrm{tot}} = \tfrac{1}{2}$ or $\tfrac{3}{2}$. $I_{\mathrm{tot}} = \tfrac{1}{2}$ gives the nucleons while $I_{\mathrm{tot}} = \tfrac{3}{2}$ forms the Δ. It is useful to consider the symmetry of the quarks inside these baryons. The internal wavefunction can be written as a product of terms, $\Psi = \psi_{\mathrm{spin}}\psi_{\mathrm{space}}\psi_{\mathrm{isospin}}\psi_{\mathrm{colour}}$, and must be antisymmetric overall under interchange (as quarks are fermions). ψ_{colour} is *always* antisymmetric (as hadrons are colourless); the symmetry of ψ_{space} is given by $(-1)^l$ and l the orbital angular momentum is zero for the long-lived hadrons we consider, so ψ_{space} is symmetric. Thus the product $\psi_{\mathrm{spin}}\psi_{\mathrm{isospin}}$ must be symmetric, implying either both must be symmetric or both must be antisymmetric. This explains the correlation between allowed spin and isospin states for the baryons: the Δ has $I = \tfrac{3}{2}$ and $s = \tfrac{3}{2}$ (both symmetric), while the nucleons have $I = \tfrac{1}{2}$ and $s = \tfrac{1}{2}$ (both antisymmetric).

In strong interactions, the total isospin vector (as well as I_3) is conserved. This is **not** true in electromagnetic or weak interactions. The conservation of isospin has observable effects on the relative rates of strong interactions, as will be discussed in the lectures.

Strangeness

It was observed that some unstable particles produced in strong interactions had a long lifetime. This unusual stability for strongly interacting particles led to the term of *strangeness*. Such particles are always produced in pairs (associated production), and the quantum number of strangeness, *S*, was introduced, which is conserved in strong interactions. Thus in the interaction $\pi^- \, p \rightarrow \Lambda^0 \, K^0$, the Λ is assigned $S = -1$ and the K has $S = +1$. These strange particles can only decay by the weak interaction, which does not conserve strangeness (as we will discuss later).

Isospin multiplet	***B***	***S***	***I***	**<*Q*>/*e***	***Y = B+S***
π^+ π^0 π^-	0	0	1	0	0
p n	1	0	½	½	1
Δ^{++} Δ^+ Δ^0 Δ^-	1	0	$\frac{3}{2}$	½	1
Λ	1	–1	0	0	0
Σ^+ Σ^0 Σ^-	1	–1	1	0	0
K^+ K^0	0	1	½	½	1
$\overline{K^0}$ K^-	0	–1	½	–½	–1
Ξ^0 Ξ^-	1	–2	½	–½	–1
Ω^-	1	–3	0	–1	–2

Table 1 A selection of strange and non-strange baryon and meson multiplets.

The formula for electric charge must know be modified to read

$$Q = e\left(I_3 + \tfrac{1}{2}B + \tfrac{1}{2}S\right) = e\left(I_3 + \tfrac{1}{2}Y\right)$$

where $Y = B + S$ is known as the hypercharge. (This formula is known as the Gell-Mann Nishijima relation.) Families of particles with similar properties (e.g. same spin and parity) can be plotted in terms of *Y* versus I_3, and form regular geometrical patterns (see figures 9 to 12):

mesons with spin-parity (J^P) = 0^- form an octet;
mesons with $J^P = 1^-$ form a nonet;
baryons with $J^P = \frac{1}{2}^+$ form an octet;
baryons with $J^P = \frac{3}{2}^+$ form a decuplet.

The difference in shape (i.e. the allowed combinations of quarks) between the baryon octet and decuplet is entirely a consequence of symmetry constraints, as will be discussed in the lectures (and below).

In terms of quarks we can introduce a new flavour of quark, the strange quark s. This has charge $-\frac{1}{3}$ and baryon number $\frac{1}{3}$ (like a d quark) but $I = 0$ and $S = -1$. It is also somewhat heavier than the u and d quarks. Since baryons consist of qqq, it is clear why no positive baryons exist with $|S| > 1$, while negative baryons are found with $S = -2$ or -3.

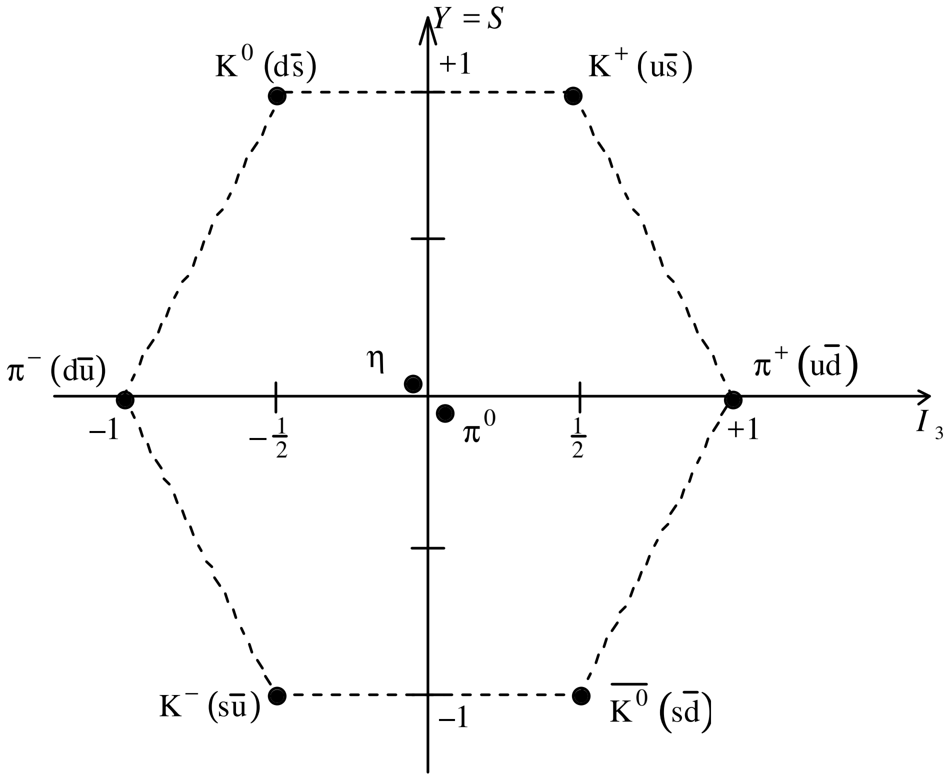

Fig. 9 The lowest-lying pseudoscalar-meson states ($J^P = 0^-$), with quark assignments indicated.
(The states at the origin are displaced slightly for clarity.)

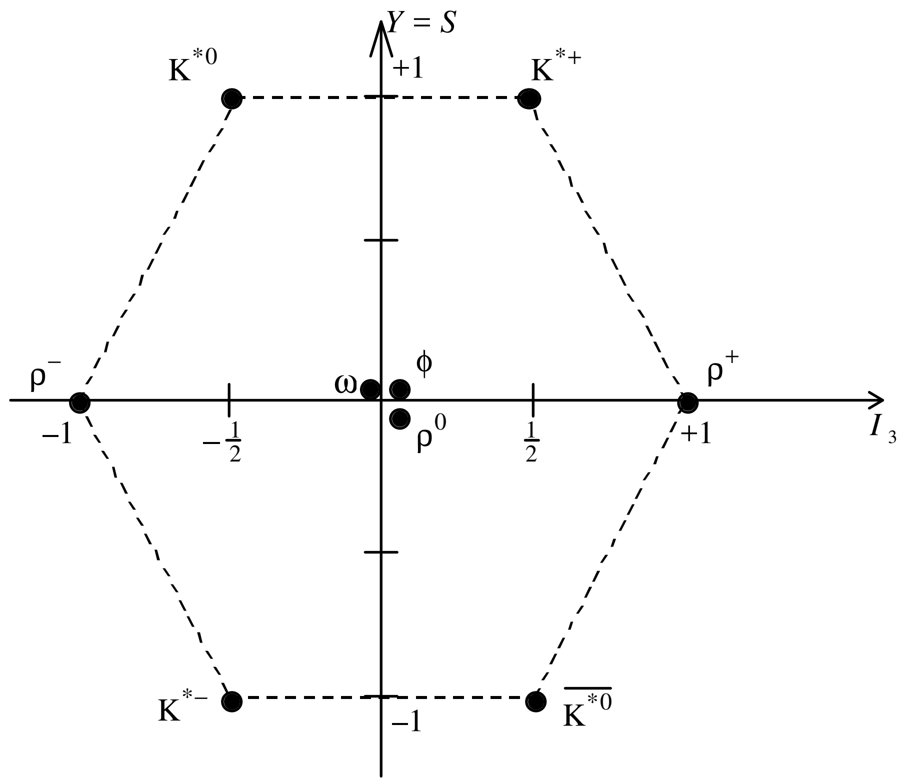

Fig. 10 The vector-meson nonet ($J^P = 1^-$).
(Quark assignments are the same as above.)

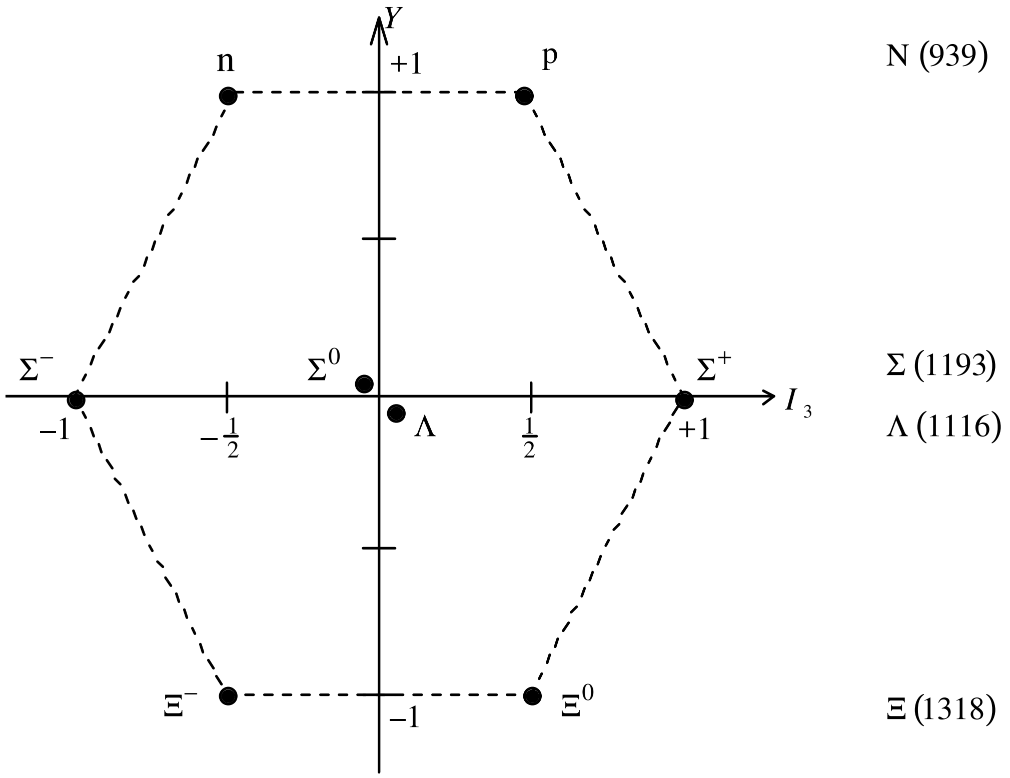

Fig. 11 The baryon octet of spin-parity $J^{\mathrm{P}} = \frac{1}{2}^{+}$ (with masses in MeV/c^2).

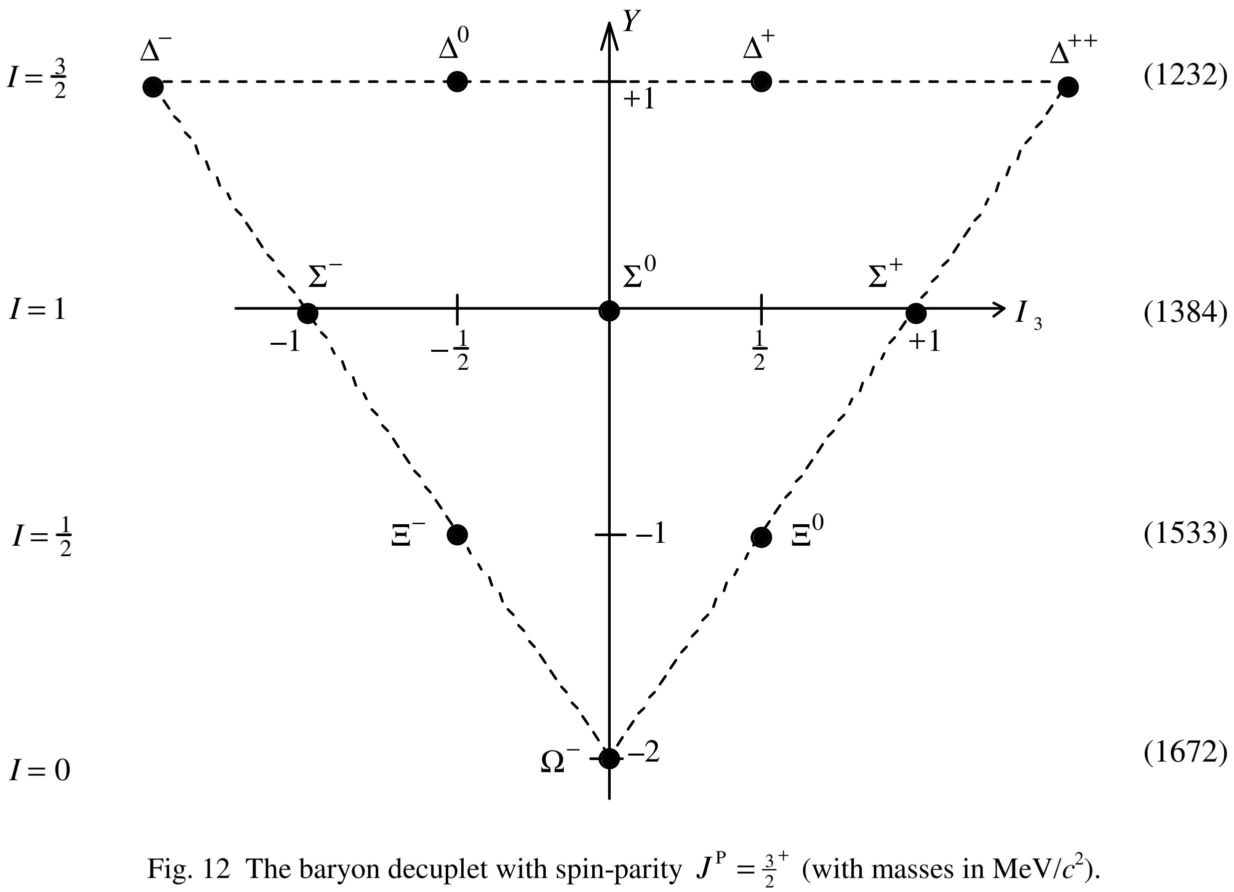

Fig. 12 The baryon decuplet with spin-parity $J^{\mathrm{P}} = \frac{3}{2}^{+}$ (with masses in MeV/c^2).

Further quarks

Other, still heavier quarks also exist. The <u>charm</u> quark, c, has a charge of ⅔, like the u, and can be considered as a partner to the s. In 3 dimensions (see figure 13) particles containing c quarks can be plotted, and again show regular patterns.

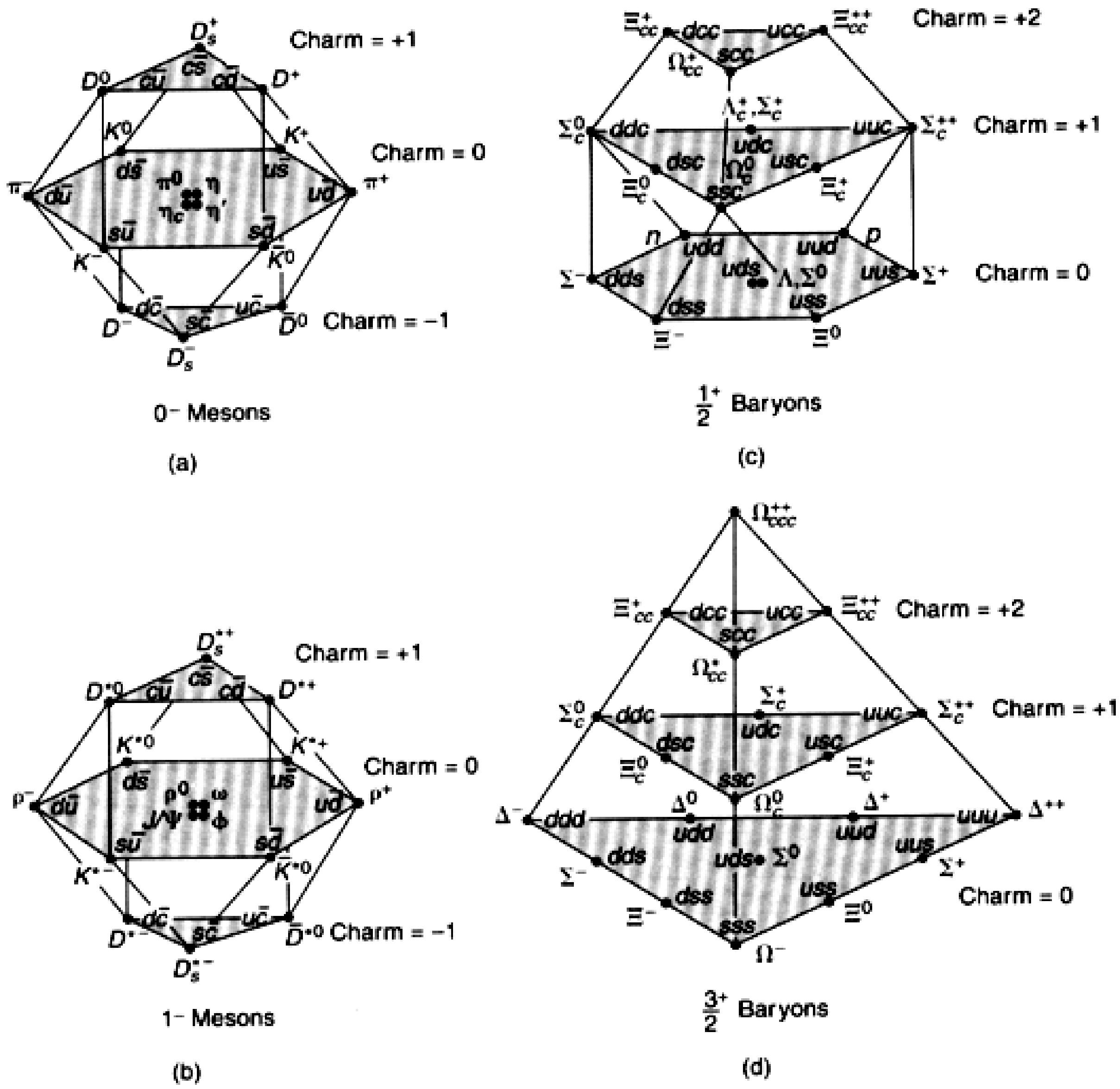

Fig. 13 Multiplets of hadrons containing up, down, strange and charm quarks. The slices through these figures where charm = 0 correspond to the plane figures already shown in the previous diagrams, though containing new particles in the case of the mesons composed of $c\bar{c}$.

We thus have 2 doublets or <u>generations</u> of quarks – (d, u) and (s, c). Since there are 3 doublets of leptons, there are theoretical reasons for expecting a third doublet of quarks too. Particles containing <u>b</u> quarks (bottom or beauty) were discovered in 1977. The b is an even heavier version of the d. Its partner, the <u>t</u> (top or truth) was first seen in 1994, and it has the greatest mass of any known fundamental particle at 174 GeV/c^2.

Flavour	Charge/e	B	I	I_3	S	c	b	t	Mass (GeV/c^2)
d	$-\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	0	0.005
u	$+\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{2}$	$+\frac{1}{2}$	0	0	0	0	0.002
s	$-\frac{1}{3}$	$\frac{1}{3}$	0	0	–1	0	0	0	0.095
c	$+\frac{2}{3}$	$\frac{1}{3}$	0	0	0	+1	0	0	1.3
b	$-\frac{1}{3}$	$\frac{1}{3}$	0	0	0	0	–1	0	4.2
t	$+\frac{2}{3}$	$\frac{1}{3}$	0	0	0	0	0	+1	174

Table 2 Quark quantum numbers and masses.

Note the convention that quarks with a negative electric charge carry a negative flavour quantum number.

The masses quoted are "bare masses" – when bound in a hadron the effective masses differ, especially for the lightest quarks. (Binding and kinetic energies mean that the u and d quarks can be treated as effectively equal in mass.)

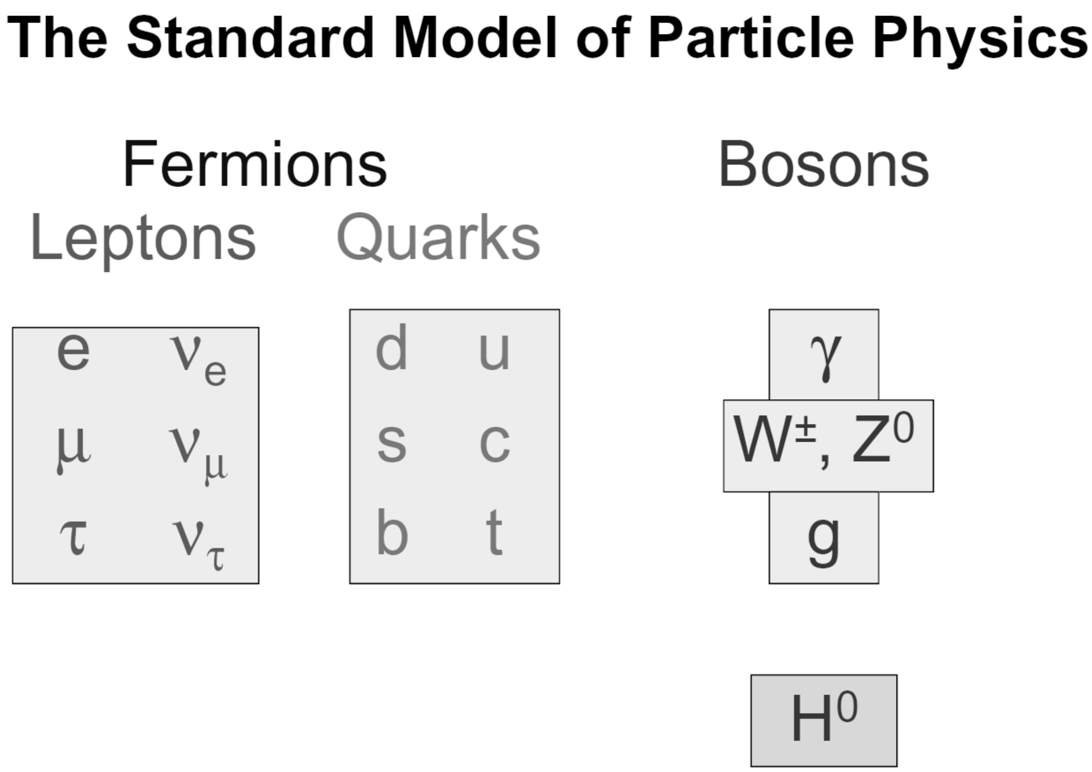

Fig. 14 The particle content of the Standard Model of Particle Physics (including the Higgs boson which is not covered in this course).

Baryons and Quark Symmetry

We considered above how 3 quarks, with spin (s) ½ and isospin (I) ½, can combine to make a total wavefunction (space, spin, isospin and colour parts) which is antisymmetric under the interchange of two particles, as required for fermions. For the lowest lying ($l = 0$) baryons, we saw that the product of the spin and isospin wavefunctions must then be symmetric, and the two solutions are the Δ, with $I = \tfrac{3}{2}$ and $s = \tfrac{3}{2}$, and the nucleons, with $I = ½$ and $s = ½$. There were a few simplifications in this account, so here is a more rigorous explanation. (This more detailed approach for 3 identical quarks is not required for the examination, but understanding it will help your appreciation of the role of symmetry in quark systems. You should be able to discuss symmetric and antisymmetric combinations of 2 quarks.)

First we will look at the combination of *two* objects with spin ½ and isospin ½. Since two quarks do not form a bound state, it is helpful to consider combinations of two nucleons as these have similar quantum numbers. In this case, since there is no colour part to the total wavefunction, the product of spin and isospin states must be *antisymmetric*. We will denote $s_z = ½$ by ↑ and $s_z = –½$ by ↓, and $I_3 = ½$ by p and $I_3 = –½$ by n.

First consider just the isospin. There are 4 combinations of two nucleons: pp, pn, np, nn. The first and last are obviously symmetric under interchange; the other 2 do not have a defined symmetry. We can produce the following combinations:

pp, $\frac{1}{\sqrt{2}}(\mathrm{pn}+\mathrm{np})$, nn, $\frac{1}{\sqrt{2}}(\mathrm{pn}-\mathrm{np})$. The first 3 are symmetric, corresponding respectively to $(I, I_3) = (1, 1)$, $(1, 0)$, $(1, –1)$. The last is antisymmetric, $(I, I_3) = (0, 0)$. Similarly the spin states are ↑↑, $\frac{1}{\sqrt{2}}(\uparrow\downarrow+\downarrow\uparrow)$, ↓↓, $\frac{1}{\sqrt{2}}(\uparrow\downarrow-\downarrow\uparrow)$, corresponding to $(s, s_z) = (1, 1)$, $(1, 0)$, $(1, –1)$, $(0, 0)$ respectively. (The $\frac{1}{\sqrt{2}}$ factors are simply the normalisations.)

The allowed states are those where spin and isospin have opposite symmetry; the antisymmetric isospin state and symmetric spin states ($s = 1$) give the bound deuteron, while the complementary states are only allowed for free pairs of protons and neutrons. (This is discussed further in the Nuclear Physics course next semester.)

We now return to the quarks, denoting $I_3 = ½$ by u and $I_3 = –½$ by d. Since the colour part of the wavefunction is always antisymmetric, the combined isospin×spin must be *symmetric*. First we will consider symmetric isospin combinations of the three quarks.

uuu is $(I, I_3) = (\tfrac{3}{2}, \tfrac{3}{2})$ so must be the Δ^{++}.

uud is $I_3 = ½$; forming combinations which are symmetric under 2-quark permutations we get $\frac{1}{\sqrt{3}}(\mathrm{uud} + \mathrm{udu} + \mathrm{duu})$. This must be $(I, I_3) = (\tfrac{3}{2}, \tfrac{1}{2})$ so the Δ^{+}.

Similarly $\frac{1}{\sqrt{3}}(\mathrm{udd} + \mathrm{dud} + \mathrm{ddu})$ is $(I, I_3) = (\tfrac{3}{2}, -\tfrac{1}{2})$ so the Δ^{0}.

ddd is $(I, I_3) = (\tfrac{3}{2}, -\tfrac{3}{2})$ so the Δ^{-}.

Note: The first and last state *must* be $I = \tfrac{3}{2}$. There must therefore be symmetric states with $I = \tfrac{3}{2}$ and intermediate I_3 values, and the above combinations are the only possibilities!

The spin states for the Δ must also be symmetric, and we can write these down in an analogous way:

$(s, s_z) = (3/2, 3/2)$ is $\uparrow\uparrow\uparrow$. $(s, s_z) = (3/2, 1/2)$ is $\frac{1}{\sqrt{3}}\left(\uparrow\uparrow\downarrow + \uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow\right)$.

$(s, s_z) = (3/2, -1/2)$ is $\frac{1}{\sqrt{3}}\left(\uparrow\downarrow\downarrow + \downarrow\uparrow\downarrow + \downarrow\downarrow\uparrow\right)$ and $(s, s_z) = (3/2, -3/2)$ is $\downarrow\downarrow\downarrow$.

It was implied earlier that the nucleon states were antisymmetric in isospin and antisymmetric in spin. In fact, it is impossible to write down 3-quark states that are antisymmetric under the interchange of *all* pairs! (Try it!) However, it is possible to use "partially antisymmetric" isospin and spin states as a basis to find the required states which are symmetric overall.

First consider one pair of quarks. Ignoring the normalisation factors, $\left(\mathrm{u}\uparrow \mathrm{d}\downarrow - \mathrm{d}\uparrow \mathrm{u}\downarrow\right)$ is antisymmetric under exchange of isospin labels; $\left(\mathrm{u}\uparrow \mathrm{d}\downarrow - \mathrm{u}\downarrow \mathrm{d}\uparrow\right)$ is antisymmetric under exchange of spin labels; so $\left(\mathrm{u}\uparrow \mathrm{d}\downarrow - \mathrm{u}\downarrow \mathrm{d}\uparrow - \mathrm{d}\uparrow \mathrm{u}\downarrow + \mathrm{d}\downarrow \mathrm{u}\uparrow\right)$ is antisymmetric under exchange of isospin or spin labels but symmetric overall (i.e. when both the spin and isospin properties of a pair of particles are swapped). This combination has $I = 0$ and $s = 0$.

We can now combine this state with a third quark $\mathrm{u}\uparrow$ and symmetrise to find the combined state representing a proton ($I_3 = 1/2$) with spin $s_z = 1/2$. Simply adding a $\mathrm{u}\uparrow$ on the left gives:
$\mathrm{u}\uparrow \mathrm{u}\uparrow \mathrm{d}\downarrow - \mathrm{u}\uparrow \mathrm{u}\downarrow \mathrm{d}\uparrow - \mathrm{u}\uparrow \mathrm{d}\uparrow \mathrm{u}\downarrow + \mathrm{u}\uparrow \mathrm{d}\downarrow \mathrm{u}\uparrow$. This must be made symmetric under interchange of pairs of quarks. Permuting first and second adds on $+\mathrm{u}\uparrow \mathrm{u}\uparrow \mathrm{d}\downarrow - \mathrm{u}\downarrow \mathrm{u}\uparrow \mathrm{d}\uparrow - \mathrm{d}\uparrow \mathrm{u}\uparrow \mathrm{u}\downarrow + \mathrm{d}\downarrow \mathrm{u}\uparrow \mathrm{u}\uparrow$, while permuting first and third adds $+\mathrm{d}\downarrow \mathrm{u}\uparrow \mathrm{u}\uparrow - \mathrm{d}\uparrow \mathrm{u}\downarrow \mathrm{u}\uparrow - \mathrm{u}\downarrow \mathrm{d}\uparrow \mathrm{u}\uparrow + \mathrm{u}\uparrow \mathrm{d}\downarrow \mathrm{u}\uparrow$. (Note that interchange of 2nd and 3rd quarks was already considered above).

Gathering terms together, and including the normalisation factor (equal as usual to the reciprocal of the sum of the squares of the coefficients) gives

$$\begin{aligned}\mathrm{p}\uparrow = \frac{1}{\sqrt{18}}(&2\mathrm{u}\uparrow \mathrm{u}\uparrow \mathrm{d}\downarrow - \mathrm{u}\uparrow \mathrm{u}\downarrow \mathrm{d}\uparrow - \mathrm{u}\downarrow \mathrm{u}\uparrow \mathrm{d}\uparrow \\ &+ 2\mathrm{u}\uparrow \mathrm{d}\downarrow \mathrm{u}\uparrow - \mathrm{u}\uparrow \mathrm{d}\uparrow \mathrm{u}\downarrow - \mathrm{u}\downarrow \mathrm{d}\uparrow \mathrm{u}\uparrow \\ &+ 2\mathrm{d}\downarrow \mathrm{u}\uparrow \mathrm{u}\uparrow - \mathrm{d}\uparrow \mathrm{u}\downarrow \mathrm{u}\uparrow - \mathrm{d}\uparrow \mathrm{u}\uparrow \mathrm{u}\downarrow).\end{aligned}$$

The spin-down state of the proton and the spin states of the neutron can be found in the same way.

Quark Flavour and the Weak Interaction

As we have already seen, the strong and electromagnetic interactions conserve quark flavour, whereas the weak interaction may change it. In many weak decays, the changes are within a generation, e.g. in beta decay the W couples a u to a d quark; in the decay $D^+ \rightarrow \overline{K}^0 \pi^+$ it couples a c to an s. However, this is not always the case, e.g. in the decay $K^- \rightarrow \pi^0 e^- \bar{\nu}_e$ the W couples an s to a u quark, and it was observed that such strangeness-changing decays were slightly weaker than strangeness-conserving weak decays.

Cabibbo explained this by proposing that the eigenstates of the weak interaction are different from those of the strong interaction. The strong interaction eigenstates are the u, d, s, c, b and t quarks, with well-defined isospin, strangeness etc. The eigenstates of the weak interaction, which does not conserve *I*, *S* etc., are said to be those of "weak isospin" *T*. For simplicity, let us first consider the first 2 generations alone. The weak eigenstates are the leptons and orthogonal linear combinations of the familiar quarks

$$\begin{Bmatrix} T_3 = +\frac{1}{2} \\ T_3 = -\frac{1}{2} \end{Bmatrix} \qquad \begin{pmatrix} \nu_e \\ e \end{pmatrix} \qquad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \qquad \begin{pmatrix} u \\ d_c \end{pmatrix} \qquad \begin{pmatrix} c \\ s_c \end{pmatrix}$$

with
$$d_c = \alpha\, d + \beta\, s$$
$$s_c = -\beta\, d + \alpha\, s \qquad \text{(normalisation } \alpha^2 + \beta^2 = 1)$$

α is usually known as $\cos\theta_c$, where θ_c is the Cabibbo angle. A value of $\sin\theta_c = 0.25$ is consistent with the observed apparent variation of weak coupling constant with reaction type. (Note that by convention it is only the $T_3 = -½$ member of the doublet which is mixed.)

The relationship between weak and strong eigenstates in 2 generations can also be expressed as

$$\underset{\textit{weak e-states}}{\begin{pmatrix} d_c \\ s_c \end{pmatrix}} = \underset{\textit{mixing matrix}}{\begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix}} \underset{\textit{strong e-states}}{\begin{pmatrix} d \\ s \end{pmatrix}}$$

The same formalism can be used for 3 generations, and the mixing matrix, known as the Cabibbo-Kabayashi-Maskawa or CKM matrix, can be parameterised in a number of ways.

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \underline{\underline{\mathbf{M}}} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

The magnitudes of the matrix elements have been determined experimentally, and are given by

$$\underline{\underline{\mathbf{M}}} = \begin{pmatrix} 0.9743 \pm 0.0002 & 0.2254 \pm 0.0006 & 0.0036 \pm 0.0002 \\ 0.2252 \pm 0.0006 & 0.9734 \pm 0.0002 & 0.041 \pm 0.001 \\ 0.0089 \pm 0.0003 & 0.040 \pm 0.001 & 0.99914 \pm 0.00005 \end{pmatrix}$$

Note that the values along the leading diagonal are quite close to one, those adjacent to it are significantly smaller, and the elements in the top-right and bottom-left corners are **much** smaller. This means that the mixing results in states which contain a small admixture of the quark from the next generation, while mixing between 1st and 3rd generation quarks is extremely small.

Flavour-changing weak interactions **always** occur via the charged current. That is, they always involve transitions between the two members of the same weak isospin doublet, e.g. between c and s′ (in either direction), or between u and d′. The mixing of the negative quarks plays a role for both initial and final state quarks. For example, the decay of a c quark is always to an s′ weak eigenstate, which will be bound in a hadron as one of the strong eigenstates of which it can be considered a mixture. On the other hand, when a hadron containing an s quark decays, the s must be considered a mixture of d′, s′ and b′ weak eigenstates, and these decay to u, c and t respectively.

Physically, the relative probability of producing hadrons containing the respective quarks in a weak decay is determined by the elements of the CKM matrix. For example, when a top quark decays it produces a b′ quark. This is bound in a hadron by the strong interaction, so must be revealed as a strong eigenstate. The b′ is most likely to result in a particle containing a b quark, with a smaller probability of an s quark and almost negligible likelihood of producing a d quark. Therefore, the near-diagonal structure of the CKM matrix means that weak decays are most likely to be <u>within a generation</u> if allowed by conservation of energy (a particle cannot decay into one that is heavier) or <u>to the next generation below</u> if this is not allowed. The most likely <u>overall</u> decay chain of a b quark is therefore $b \to c \to s \to u$.

When the charged-current weak decays are considered along with binding into strong eigenstates in the hadrons, the elements of $\underline{\underline{\mathbf{M}}}$ can be interpreted as giving the effective transition strengths between quarks as follows:

$$\underline{\underline{\mathbf{M}}} = \begin{pmatrix} u \leftrightarrow d & u \leftrightarrow s & u \leftrightarrow b \\ c \leftrightarrow d & c \leftrightarrow s & c \leftrightarrow b \\ t \leftrightarrow d & t \leftrightarrow s & t \leftrightarrow b \end{pmatrix}.$$

(Again, physical decays must always be to the lighter quark.)

For two generations, one parameter was required to describe the mixing. This was the Cabibbo angle. With three generations, 4 independent parameters are needed to define a general unitary matrix, and the individual matrix elements may have imaginary parts. One possible parameterisation of the CKM matrix is given below. Note that the following material is provided for completeness only, and is <u>**not examinable**</u>! (Further details are provided in the text books.)

$$\underline{\underline{\mathbf{M}}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$, with i and j being generation labels $\{i,j = 1,2,3\}$. In the limit $\theta_{23} = \theta_{13} = 0$, the third generation decouples, and the situation reduces to the usual Cabibbo mixing of the first two generations, with θ_{12} identified with the Cabibbo angle. The real angles θ_{12}, θ_{23}, θ_{13} can all be made to lie in the first quadrant by suitable definition of the quark field phases. c_{13} is known to differ from unity only in the sixth decimal place.

If the parameter δ is non-zero, then the matrix is complex, and the small degree of CP violation present in the weak interaction can be explained naturally. This has not yet been conclusively proven!

[The above parameterisation and values are taken from the Particle Physics Data Booklet, from "Review of Particle Physics", Chinese Physics **C38**, July 2014, by the Particle Data Group.]

The K^0 System and Strangeness Regeneration

We have seen that the strong and weak eigenstates, when expressed in terms of quarks, are different. We now look at evidence for different eigenstates in the observed, free particles themselves. This is seen in the neutral kaon system.

The $K^0\left(d\bar{s}\right)$ has strangeness 1, and can be produced in π p collisions by the strong interaction in association with a Λ hyperon (i.e. strange baryon). In contrast, the $\overline{K^0}\left(s\bar{d}\right)$ has strangeness –1, and as there are no baryons of positive strangeness, this can only be formed in higher energy collisions. A pure sample of K^0 can therefore be produced.

The weak interaction does not conserve strangeness. It therefore does not distinguish between K^0 and $\overline{K^0}$. The eigenstates of the weak interactions are not those of strangeness but (approximately) those of CP. It will be shown in the lecture that these can be written as

$$K_1 = \tfrac{1}{\sqrt{2}}\left(\left|K^0\right\rangle - \left|\overline{K^0}\right\rangle\right) \qquad \text{with CP e-value } +1$$

$$K_2 = \tfrac{1}{\sqrt{2}}\left(\left|K^0\right\rangle + \left|\overline{K^0}\right\rangle\right) \qquad \text{with CP e-value } -1$$

The K_1 decays to two pions, while the K_2 must decay to a 3-pion final state. As there is much more phase space available for the K_1 decay, its decay rate is about 600 times greater than that for the K_2.

Consider a beam of pions striking a thin solid target, mounted in a vacuum. Strong interactions will occur, resulting in the production of K^0 (with no $\overline{K^0}$). These then travel on and decay through the weak interaction. The K^0 is equivalent to an equal mixture of K_1 and K_2, and the K_1 component rapidly decays away. If a further solid target (known as a regenerator) is introduced some distance downstream, the remaining kaons will interact with it via the strong interaction. Here the surviving K_2 component must be seen as an equal mixture of K^0 and $\overline{K^0}$, so $S = +1$ and -1 states will be produced, even though only K^0 (with $S = +1$) was present initially! In fact, the $\overline{K^0}$ will be preferentially absorbed, with only some of the K^0 emerging from the regenerator. Here, weak decays will again occur, and an equal mixture of K_1 and K_2 will again be observed, even though previously the entire K_1 component had decayed away!

Quantum mechanically, the situation is exactly equivalent to a series of Stern-Gerlach experiments, with crossed magnetic field gradients. If an unpolarised beam of neutral, spin-½ atoms travelling along the z-axis encounters a region with a field gradient in the x-direction, it will split into two equal divergent beams with $s_x = \pm½$. If one of these is blocked and the other allowed into a second region where the field gradient is in the y-direction, it will split again into equal divergent beams with $s_y = \pm½$. If now one of these is selected and encounters a region with a field gradient again in the x-direction, it will split into two equal beams with $s_x = \pm½$ even though one of these components had previously been eliminated.