Particle Physics – Introductory Notes

1. Units of Energy, Momentum and Mass

As in atomic and solid state physics, a useful unit of energy in particle and nuclear physics is the electron volt. This is the amount of kinetic energy gained by an electron when it is accelerated through a potential difference of one volt. Normally the energies involved in nuclear reactions are millions of electron volts (MeV) and in high energy particle interactions they may be thousands of millions of electron volts or Giga electron volts (GeV = 10^9 eV). A convenient unit for particle masses makes use of the Einstein mass-energy relationship

$$E = m\, c^2.$$

This yields a unit for mass expressed as energy divided by the square of the velocity of light, MeV/c^2 or GeV/c^2. For example

$$\text{proton mass} = 938.3 \text{ MeV}/c^2$$
$$\text{electron mass} = 0.511 \text{ MeV}/c^2.$$

This system of units is extended to momentum through the relativistic relationship for the energy of a particle of rest mass m moving with momentum p,

$$E^2 = p^2c^2 + m^2c^4.$$

From this, it follows that if we express momentum in the units of energy divided by the velocity of light (GeV/c), we have a self consistent system in which the velocity of light is implicitly used, but its value does not have to be explicitly put in to the calculations. Using these units, we can write

$$E^2 \,(\text{GeV})^2 = p^2 \,(\text{GeV}/c)^2 + m^2 \,(\text{GeV}/c^2)^2.$$

Kinetic energy, T, is just expressed in GeV, so

$$E\,(\text{GeV}) = T\,(\text{GeV}) + m\,(\text{GeV}/c^2).$$

The velocity of the particle in units of the velocity of light is given by

$$\beta = \frac{p\ (\text{GeV}/c)}{E\ (\text{GeV})},$$

and the relativistic gamma factor is given by

$$\gamma = \frac{E\ (\text{GeV})}{m\ \left(\text{GeV}/c^2\right)}.$$

In the **non-relativistic** limit $\beta \to 0$ we have $p \ll m$. Thus the expression for kinetic energy reduces as follows

$$\begin{aligned} T &= E - m \\ &= (p^2 + m^2)^{1/2} - m \\ &= m\,(1+p^2/m^2)^{1/2} - m \\ &\approx p^2/2m \quad \left(\text{or } \tfrac{1}{2}mv^2\right) \qquad \text{as expected.} \end{aligned}$$

2. Cross-sections and Decay Rates

The idea of cross-section arises from the simplest model of a nucleus (or some other particle) as a completely absorbing sphere of cross-sectional area σ. Consider a uniform beam of N particles per second per unit area incident on a thin sheet of material of thickness dt, in which there are n absorbing nuclei per unit volume. The effective nuclear area for absorption is then $\sigma\, n\, \mathrm{d}t$ per unit area of sheet (assuming that $\sigma\, n\, dt \ll 1$ so that no nuclei are hidden one behind another). The rate at which particles are removed from the beam is then just

$$-\,dN = N\,\sigma\, n\, dt \quad (s^{-1})$$

so the intensity of a beam passing through a thick sheet will decrease exponentially with distance t into the target

$$N(t) = N_0 \exp(-\,\sigma\, n\, t).$$

This simple model in which the probability of absorption, or some other interaction, is unity within a certain radius of the centre of a nucleus and zero elsewhere does not correspond with physical reality, but nevertheless the cross-section σ is a very useful way of expressing the overall probability per nucleus (or other target particle) that a given interaction will occur.

The unit of cross-section used in nuclear and particle interactions is the barn, b, equal to 10^{-28} m^2. In interactions between high energy particles, smaller units such as the millibarn (10^{-3} b) or even picobarn (pb $=10^{-12}$ b) are often used.

In most cases there are several possible reactions between the incident and target particles, and the cross-section for each will be different. These individual cross-sections are known as partial cross-sections, and their overall sum is the total cross-section.

After a reaction or scattering has occurred the outgoing particles often have an anisotropic distribution, with different energies at different directions. Then the number of particles scattered per second into solid angle $d\Omega$ at (θ,ϕ) with respect to the incoming beam is written

$$N_0\, \sigma\, n\, t\, f(\theta,\phi)\, d\Omega$$

where $\int f(\theta,\phi)\, d\Omega = 1$ and $f(\theta,\phi)$ is the angular distribution for the process. The product $\sigma f(\theta,\phi)$ is usually written as a single function $\frac{d\sigma(\theta,\phi)}{d\Omega}$ – the differential cross-section. The partial cross-section for the process can be obtained by integrating the differential cross-section over all solid angles.

$$\sigma = \int_0^{2\pi}\int_0^{\pi} \frac{d\sigma}{d\Omega} \sin(\theta)\, d\theta\, d\phi .$$

Very often there is no dependence on ϕ and the integral reduces to

$$\sigma = 2\pi \int_0^{\pi} \frac{d\sigma}{d\Omega} \sin(\theta)\, d\theta .$$

Consider the interaction

$$a + A \rightarrow B + b$$

where a beam of particles of type a strikes a target of type A.

Per unit target particle (A), the transition rate, or interaction rate, is just $\sigma\, \Phi$, where Φ is the flux of incident particles (a). Thus a useful definition of the cross-section for the interaction is the transition rate **W** per unit incident flux per target particle. In quantum mechanics, the value of **W** is given by the product of the square of the matrix element M_{fi} between initial (i) and final (f) states and a density of final states or phase space factor D_f. The matrix element contains all the dynamical features of the interaction such as its strength, energy dependence and angular distribution.

$$\mathbf{W} = \frac{2\pi}{\hbar} |M_{fi}|^2 D_f$$

This formula is often known as Fermi's Golden Rule, and is applicable to nuclear reactions, decay processes, atomic transitions, scattering processes, etc.

3. Kinematics of Nuclear and Elementary Particle Reactions

When we discuss a reaction, we normally assume that in the initial state the incoming particle and target are well separated and so non-interacting, and that this is also true of the produced particles in the final state. Thus there are no forces between them and no potential energy term in the expression for the total energy,

$$E = T + m \quad \text{or} \quad E^2 = p^2 + m^2.$$

(Here again, the momenta and masses are expressed in units of energy/c and energy/c^2 respectively.)

It is very often useful to consider processes in different **frames of reference**, having relative motion between them. Note that these are just different ways of describing a given process. If a reaction occurs or is allowed in one frame, then it occurs or is allowed in any other. Note also that frames of reference are not specifically a feature of special relativity. Individual kinematic properties of particles (such as velocity, energy or momentum) will obviously be different in different frames. Although a calculation can be carried out in any frame, some frames are particularly simple. For a single particle, the **rest frame** is often useful. Here there is no motion so $E = m$ and $p = 0$. For a system of particles, the **centre-of-mass** (or **C of M**) frame may be appropriate. This is the frame moving with the velocity of the centre of mass of the system. Although individual particles (in general) have non-zero momentum in this frame, the vector sum of momenta must be zero, $\Sigma_i \underline{p}_i = 0$. (This frame is therefore also sometimes known as the "zero momentum frame".) At the end of a calculation, kinematic properties normally have to be returned in the laboratory frame, in which the problem was originally posed.

For processes involving elementary particles, it is usually <u>not</u> reasonable to use non-relativistic approximations. As a general example of calculations involving kinematic quantities, consider the motion of any group of (non-interacting) particles with individual energies E_i and momenta $\underline{p}_i$. This motion can be divided into two components:

i) the absolute motion of the centre of mass of the group, and
ii) the relative motion of each particle with respect to that centre of mass.

The total energy and momentum are just

$$E = \Sigma_i E_i \qquad\qquad \underline{p} = \Sigma_i \underline{p}_i.$$

The *effective* rest mass of the group W is defined by

$$W^2 = \left(\sum_i E_i\right)^2 - \left|\sum_i \underline{p}_i\right|^2.$$

The absolute velocity of the centre of mass of the group in the laboratory is

$$\beta_{cm} = |\underline{p}|/E, \qquad \text{and} \qquad \gamma_{cm} = E/W = \left(1-\beta_{cm}^2\right)^{-1/2}.$$

W, just like the rest mass of a single particle, is independent of the relative motion of the observer. For example, if the observer is moving with a velocity β along the z axis (in the laboratory), then, employing Lorentz transformations, the observed total energy and momentum of the particles will be

$$E' = \gamma E - \gamma\beta p_z$$
$$p_z' = \gamma p_z - \gamma\beta E$$
$$p_y' = p_y$$
$$p_x' = p_x$$

Therefore

$$\begin{aligned}(W')^2 &= (E')^2 - |\underline{p}'|^2 \\ &= \gamma^2 E^2\left(1-\beta^2\right) - \gamma^2 p_z^2\left(1-\beta^2\right) - p_y^2 - p_x^2 \\ &= E^2 - |\underline{p}|^2 \\ &= W^2\end{aligned}$$

i.e. <u>W is invariant – the same in all frames</u>.

The above invariance can be expressed by writing $\underline{p}$, E as the components of a **four vector** with $p_1 = p_x$, $p_2 = p_y$, $p_3 = p_z$ and $p_4 = iE$. The square of the length of this 4-vector is

$$p^2 = \sum_\mu p_\mu^2 = p_1^2 + p_2^2 + p_3^2 + p_4^2 = \left|\underline{p}^2\right| - E^2 = -m^2 \qquad \text{(or } -W^2\text{)}$$

which is relativistically invariant. The use of 4-vectors can facilitate relativistic calculations of elementary particle interactions, compressing the conservation of energy and momentum into a single expression.

Note that in transforming to the centre of mass frame (i.e. that frame in which the centre of mass of the group is at rest, so that $\Sigma_i \underline{p}_i = 0$), we have

$$W = \Sigma_i E_i = \Sigma_i m_i + \Sigma_i T_i$$

The mass of the system is equal to the sum of the masses of the constituents plus the total "internal" kinetic energy of the group. Since this latter must be positive, **the mass of the system exceeds the sum of the masses of the constituents**.

Remember that in a decay or an interaction, E and $\underline{p}$ are conserved. This means that the combination $E^2 - |\underline{p}|^2$ is both *invariant* and *conserved*. It is therefore the same in any frame and at any time – before or after an event. This is a property we will make frequent use of!

4. Problem Solving for Decays and Collisions

As part of this course, we will work through increasingly sophisticated examples of particle decays and two-body reactions. There will be some illustrations in lectures, but most instances will be covered in a programme of homeworks. The physical principles employed in solving all these problems are the same: conservation of energy, conservation of momentum and (for individual particles) $E^2 - p^2 = m^2$.

In general there will be some initial or final particles for which conditions are specified – e.g. particle is at rest, or moving with a certain (total) energy – or required to be found. There are also likely to be particles whose kinematic properties are neither given nor of concern. For example, a pion decays into a muon and neutrino, and you may be asked for the energy of the muon but not for the neutrino.

The simplest problem solving strategy is then as follows.

- First, write down equations embodying conservation of energy and momentum.
- Second, eliminate from these equations the properties of the "uninteresting" particle(s), e.g. by using the relationship between E and p for these particles.
- Finally, rearrange the resulting equation to determine the required quantity. If asked for the energy of a particle, you may at this stage need to express its (unknown) momentum in terms of that energy.

Although it is possible to carry out these steps in a different order, that will generally result in mathematically more complex calculations, and may introduce additional, possibly non-physical, solutions.