Evidence in Support of the Quark Model

The quark model originally arose from the analysis of symmetry patterns using group theory. The octets, nonets, decuplets etc. could easily be explained with coloured quarks and the application of the Pauli exclusion principle. It is found that the quark model explains a large number of features of the observed particles and their interactions. However, we must consider whether quarks are mathematical abstractions, or whether there is evidence for point-like, fractionally charged, coloured constituents.

Electron-Positron Annihilation

Electron-positron annihilation to produce a particle-antiparticle pair proceeds through a time-like virtual photon. (The difference between time-like and space-like virtual particles will be explored in the lecture.) At energies much greater than twice the rest mass of a quark, the amplitude for pair production of a quark-antiquark pair is proportional to the product of the charge on an electron, e, and the charge on the quark, say $z\,e$. The cross-section is thus proportional to z^2e^4. The quarks are, of course, not observed themselves, but seen in the combinations known as hadrons. The ratio, R, of the cross-section for production of hadrons divided by that for production of $\mu^+\mu^-$ pairs at the same energy is just given by $R = \Sigma_i\, z_i^2$ where the sum is over all quarks which can take part in the production.

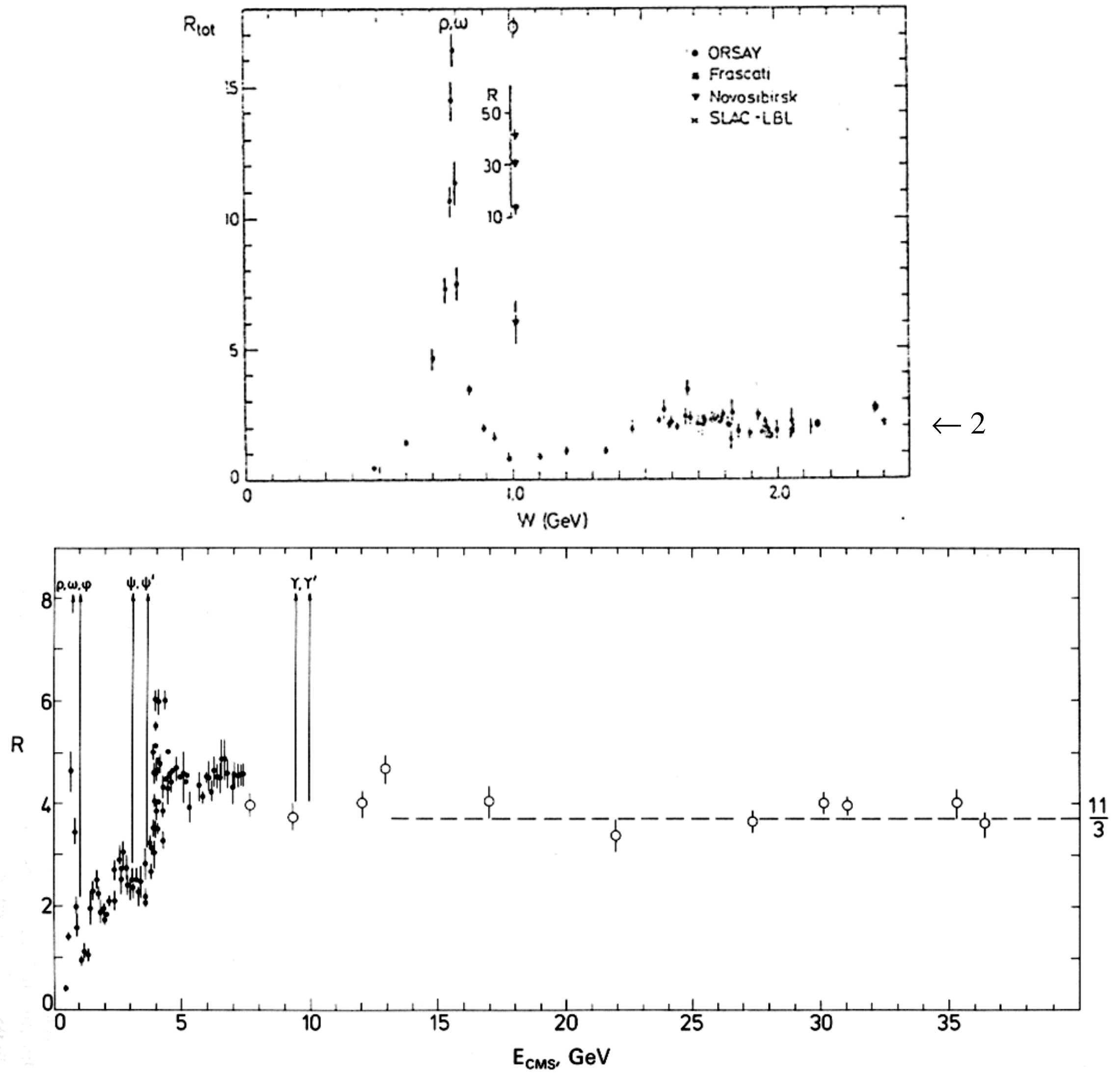

Fig. 15 The ratio R of the cross-section for $e^+e^- \to$ hadrons, divided by that for $e^+e^- \to \mu^+\mu^-$. The fact that R is constant above 10 GeV CMS energy is proof of the point-like nature of hadron constituents. The predicted value of R, assuming that the primary process is formation of a quark-antiquark pair, is 11/3 if pairs of u, d, s, c, b quarks are excited and they have three colour degrees of freedom. The data come from many storage-ring experiments.

Note that since each quark can exist in 3 colours, the sum must be over both colours and flavours. At low energies, where u, d and s quarks can be produced, the value of R is then predicted to be $R = 3\left(\left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^2\right) = 2$, compatible with the value shown in figure 15 (upper). As the c and b thresholds are crossed, the value of R goes through wide excursions in resonance regions before settling down to values expected to be $3\frac{1}{3}$ and $3\frac{2}{3}$. In fact, above 3.6 GeV the heaviest lepton, the τ, is also produced, and this decays predominantly hadronically, adding another unit to R – see figure 15 (lower). (Above the b threshold, the longer path-length of the τ allows these decays to be removed).

Deep Inelastic Lepton Scattering

The values of R above indicate that hadrons are indeed made of fractionally charged, coloured objects. We now look for evidence that these are really point-like particles, in the inelastic scattering of electrons or muons off nucleons via the exchange of a virtual photon. This occurs when the struck proton or neutron absorbs energy in breaking up to form a hadronic system, of invariant mass W. W is not always easy to measure directly, but may be deduced by considering the four-momentum transfer, q, in the scattering. We have already seen that, to a good approximation, $q^2 = 2E_i E_f(1-\cos\theta)$ where E_i and E_f are the initial and final lepton energies and θ is the lepton scattering angle. From the hadronic state's point of view, it is easy to show that $q^2 = M^2 + 2M\nu - W^2$ where M is the mass of the nucleon and ν is the change in energy of the hadronic state (and hence minus that of the lepton). Hence q^2 and ν define W.

For elastic scattering, when the nucleon remains a nucleon, $W = M$ and $q^2 = 2M\nu$. Equivalently, if we define x as $\frac{q^2}{2M\nu}$, then $x = 1$. q and ν are no longer independent, but are said to "scale". Now if we consider the nucleon to be made up of stationary point-like particles of mass m_q, then $\frac{q^2}{2M\nu}$ will be a constant for elastic scattering off *these* particles, which will fix $x = \frac{m_q}{M}$. However, unlike the nucleon, the quark will *not* be at rest, having considerable momentum within the proton. When we perform a Lorentz transform from the rest frame of a quark to that of the proton, integrating over the distribution of quark momenta leads to the form factor of the proton. However, as long as the quarks are point-like the form factor should only depend on q^2 through the dimensionless ratio x, and the cross section shows "scale invariance". This is indeed observed, as shown in figure 16. (Here F_2 is proportional to the form factor.)

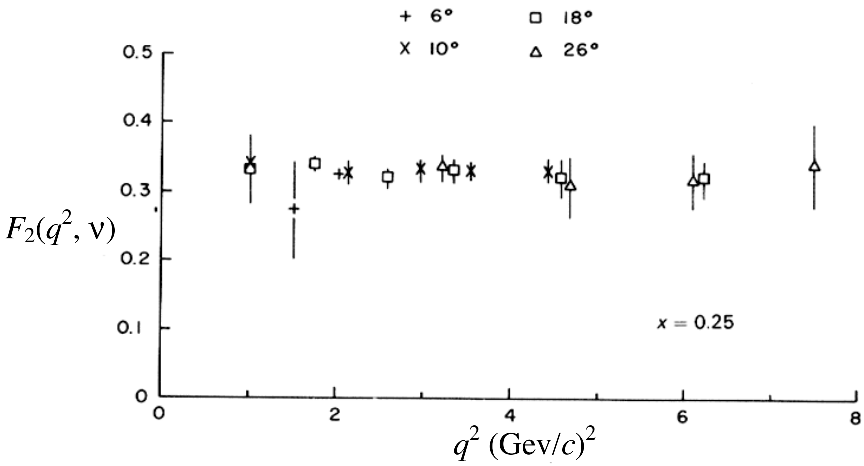

Fig. 16 $F_2(q^2, \nu)$ as a function of q^2 at $x = 0.25$. For this choice of x, there is practically no dependence on q^2, that is, there is exact "scale invariance". (Data from the Stanford Linear Accelerator Center.)

Scattering from an extended object like the proton (rather than from point-like constituents within it) would produce a very different distribution. As calculated in an early homework, the form factor for a structure-less proton drops rapidly with q^2, reaching very small values for q^2 above 1 to 2 (GeV/c)^2.

However, this is not the whole story! It is found that quarks carry only half the momentum of a moving nucleon, the rest being carried by electrically neutral gluons, which are invisible to the virtual photon. The gluons also produce virtual $q\overline{q}$ pairs, and if the probing photon has high enough energy (or q^2) it can also scatter these into real (positive energy) states. So at high enough energies, the structure functions do indeed start to depend on q^2, and scale invariance is violated!

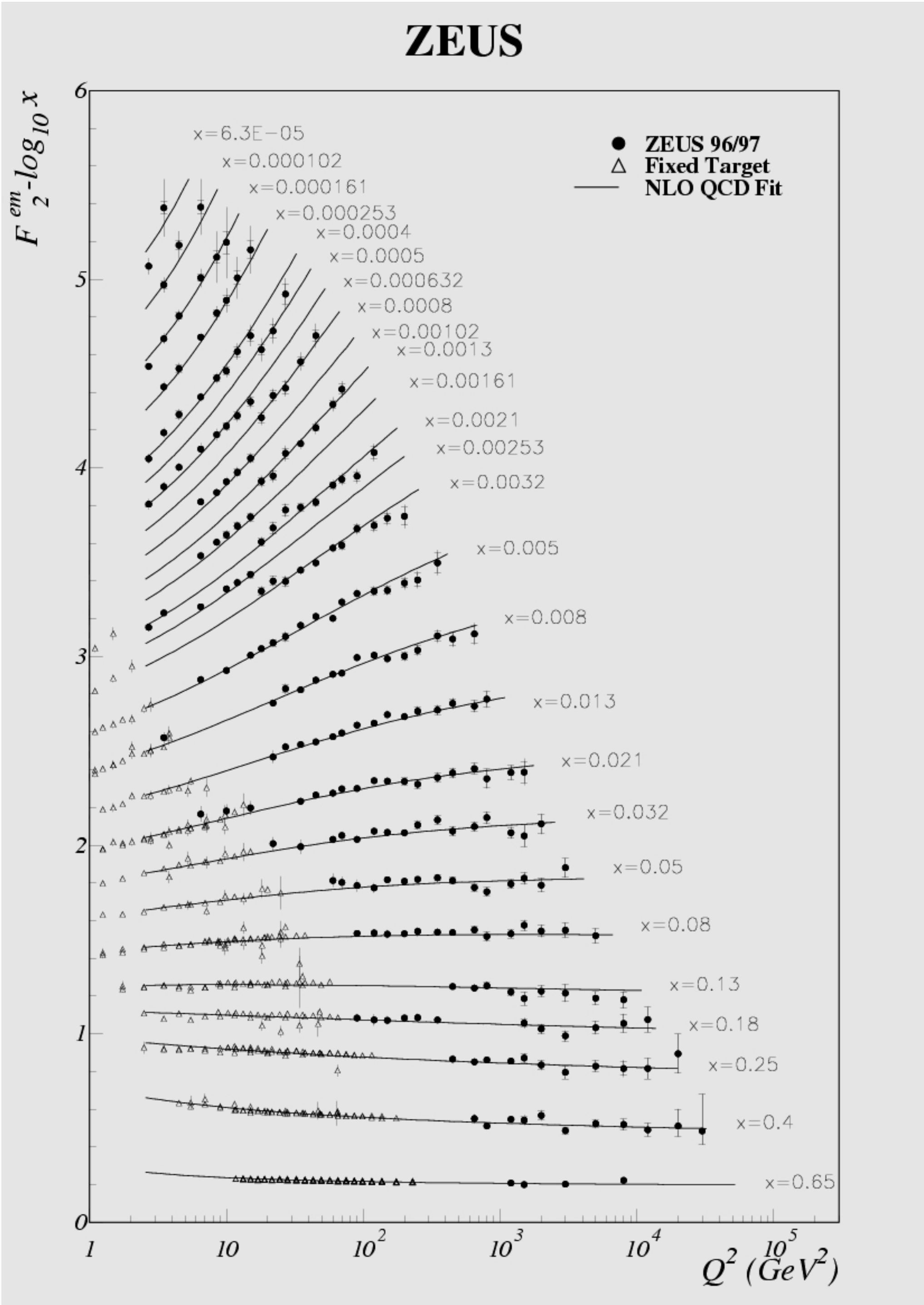

Fig. 17 Electron-proton scattering data from HERA. For x greater than about 0.1, there is little dependence of F_2 on q^2 for a given x, and scaling holds. For small x, F_2 increases with q^2 due to scattering off virtual quark-antiquark pairs, and scaling is violated. Note the variable on the ordinate axis is defined to displace the points vertically according to their x value, to prevent the lines lying on top of each other!