

Evidence for Quarks

The quark model originally arose from the analysis of symmetry patterns using group theory.

The octets, nonets, decuplets etc. could easily be explained with coloured quarks and the application of the Pauli exclusion principle.

It is found that the quark model explains a large number of features of the observed particles and their interactions.

However, we must consider whether quarks are mathematical abstractions, or whether there is evidence for **point-like, fractionally charged, coloured** constituents.

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Electron-Positron Annihilation

→ At energies much greater than twice the rest mass of a quark, the amplitude for pair production of a quark-antiquark pair is proportional to the product of the charge on an electron, e , and the charge on the quark, say $z e$.

→ The cross-section is thus proportional to $z^2 e^4$.

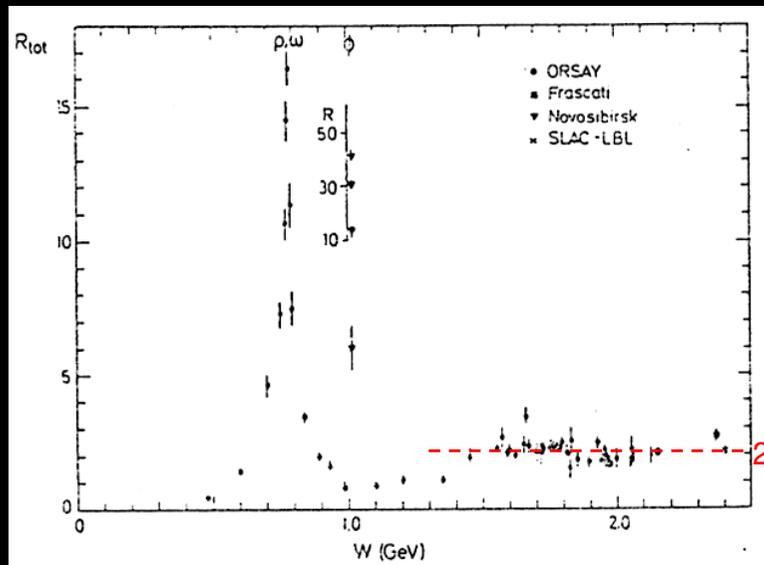
The quarks are, of course, not observed themselves, but seen in the combinations known as hadrons.

The ratio, R , of the cross-section for production of hadrons divided by that for production of $\mu^+ \mu^-$ pairs at the same energy is just given by

$$R = \sum_i z_i^2$$

where the sum is over all quarks which can take part in the production.

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The ratio R of the cross-section for $e^+e^- \rightarrow \text{hadrons}$, divided by that for $e^+e^- \rightarrow \mu^+\mu^-$.

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Note that since each quark can exist in 3 colours, the sum must be over both colours and flavours.

At low energies, where u , d and s quarks can be produced, the value of R is then predicted to be

$$R = 3 \left(\left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^2 \right) = 2$$

compatible with the value shown in the figure.

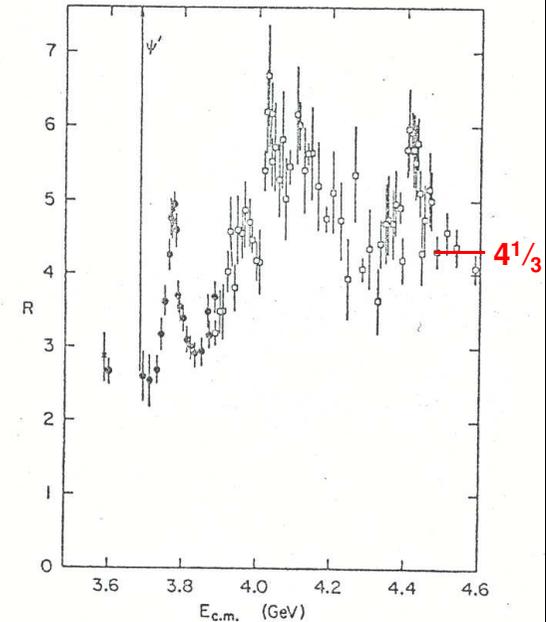
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As the c and b thresholds are crossed, the value of R goes through wide excursions in resonance regions, before settling down to values expected to be $3^{1/3}$ and $3^{2/3}$.

In fact, above 3.6 GeV the heaviest lepton, the τ , is also produced, and this decays predominantly hadronically, adding another unit to R – see the second figure.

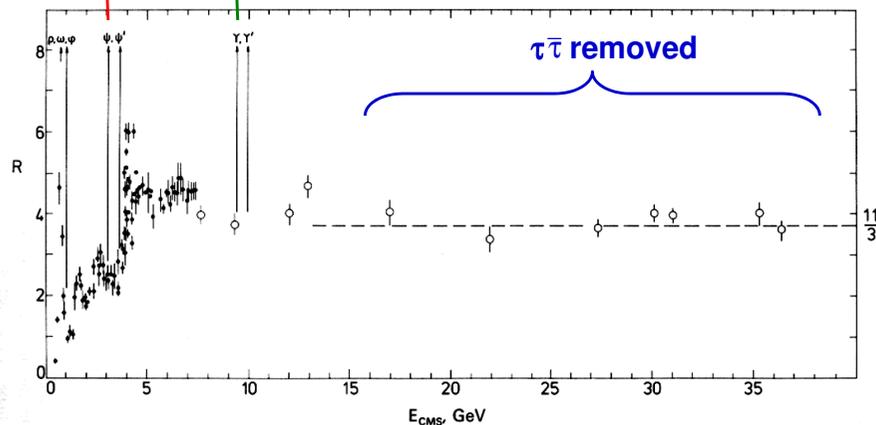
(Above the b threshold, the longer path-length of the τ allows these decays to be removed).

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c threshold ■ **b threshold**



The ratio R . The fact that R is constant above 10 GeV CMS energy is proof of the point-like nature of hadron constituents. The predicted value of R , assuming that the primary process is formation of a quark-antiquark pair, is $11/3$ if pairs of u, d, s, c, b quarks are excited and they have three colour degrees of freedom. The data come from many storage-ring experiments.

The observed values of R as the different quark thresholds are crossed are **only** compatible with particles which are:

- fractionally charged
- occurring in triplets of colour.

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Deep Inelastic Lepton Scattering

The values of R above indicate that hadrons are indeed made of **fractionally charged, coloured** objects.

We now look for evidence that these are really **point-like** particles, in the inelastic scattering of electrons or muons off nucleons via the exchange of a virtual photon.

This occurs when the struck proton or neutron absorbs energy in breaking up to form a hadronic system, of invariant mass W .

W is not always easy to measure directly, but may be deduced by considering the four-momentum transfer, q , in the scattering.

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We have already seen that, to a good approximation,

$$q^2 \approx 2E_i E_f (1 - \cos \theta)$$

where E_i and E_f are the initial and final lepton energies and θ is the lepton scattering angle.

→ From the hadronic state's point of view, we can show that

$$q^2 = M^2 + 2M\nu - W^2$$

where M is the mass of the nucleon and ν is the change in energy of the hadronic state (and hence minus that of the lepton).

Hence

q^2 and ν define W .

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For **elastic** scattering, when the nucleon remains a nucleon,

$$W = M \quad \text{and} \quad q^2 = 2M\nu.$$

Equivalently, if we define x by

$$x = \frac{q^2}{2M\nu}$$

then **$x = 1$.**

q and ν are no longer independent, but **"scale"**.

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Now if we consider the nucleon to be made up of *stationary* point-like particles of mass m_q , then $\frac{q^2}{2M\nu}$ will be a constant for elastic scattering off *these* particles, which will fix

$$x = \frac{m_q}{M}$$

However, unlike the nucleon, the quark will *not* be at rest, having momentum within the nucleon (uncertainty principle).

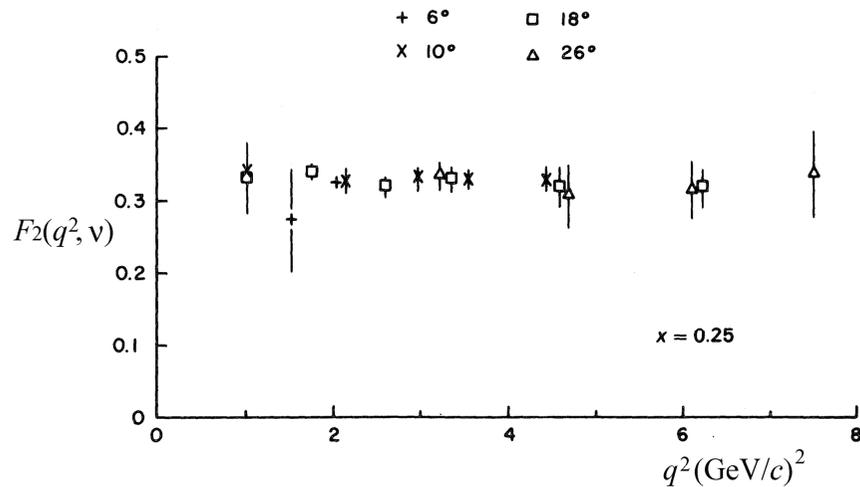
The distribution of momenta leads to a form factor for the proton, but as long as the quarks are point-like the form factor should only depend on q^2 through the dimensionless ratio x .

This is indeed observed, as shown in the figure.

(Here F_2 is proportional to the form factor.)

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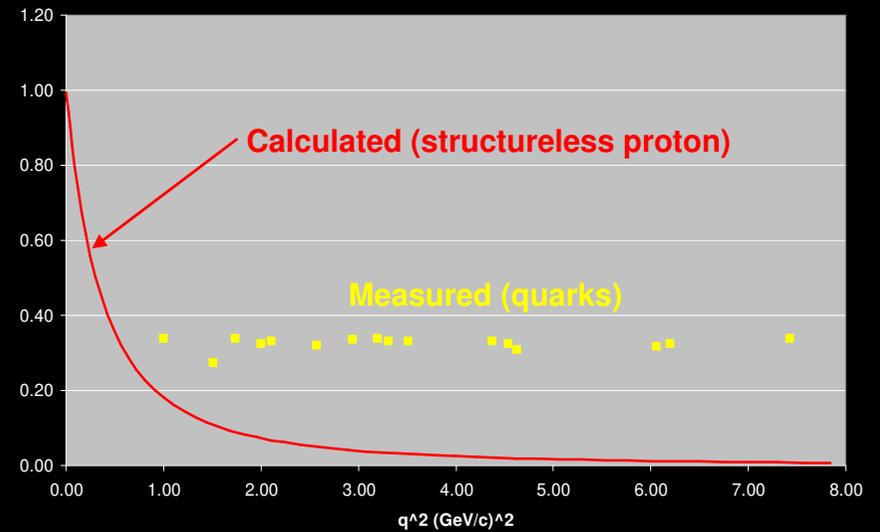
SLAC e-p scattering



$F_2(q^2, \nu)$ as a function of q^2 at $x = 0.25$. For this choice of x , there is practically no dependence on q^2 , that is, there is exact “scaling”.

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Proton Form Factor



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Scattering off quarks – Summary

- Define $x = q^2/2M\nu$
- For elastic scattering off nucleon
 - $x = 1$
- For scattering off stationary quarks, expect:
 - $x = m_q/M = \text{constant}$
 - F.F. = 1 if quarks are point-like
- From moving point-like quarks, inside nucleon:
 - $F_2(q^2, \nu) = \text{constant}$ for fixed x value
 - This is observed. Conclusion: quarks are point-like

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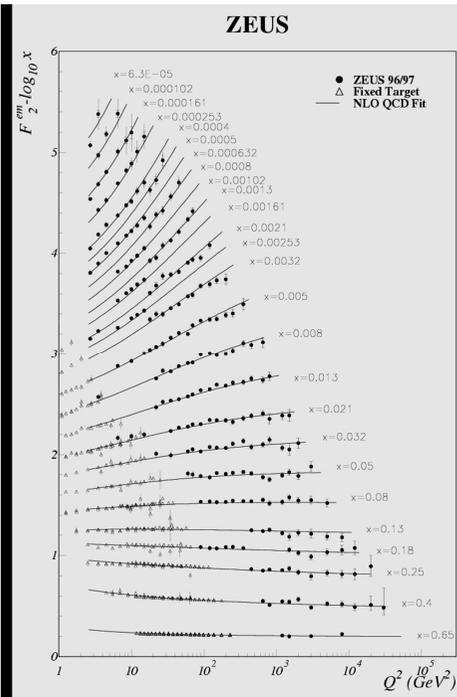
However, this is not the whole story!

It is found that quarks carry only half the momentum of a moving nucleon, the rest being carried by electrically neutral gluons, which are invisible to the virtual photon.

- The gluons also produce virtual q pairs, and if the probing photon has high enough energy (or q^2) it can also scatter these into real (positive energy) states.

So at high enough energies, the structure functions do indeed start to depend on q^2 , and scale invariance is violated!

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Results from e-p collisions at HERA.

(The data are displaced vertically according to their x value, for clarity.)

What you should have learned

- **Space-like and time-like virtual photons**
- **Variation of R with energy**
 - Evidence for colour and fractional charge
- **Deep inelastic scattering**
 - Definition of x ($x = 1$ for elastic scattering)
 - Scaling; constancy of structure function for fixed x
 - Evidence for point-like constituents
 - Scaling violation at high q^2 (and low x).