Fermions and Bosons

A multi-particle wave function for non-interacting (e.g. widely separated) particles can be written as the product of single particle functions.

$$\Psi(1,2,3,\ldots) = c\,\Psi_A(1)\,\Psi_B(2)\,\Psi_C(3)\ \ldots$$

where A, B, C describe the quantum numbers of the state and 1, 2, 3 give the co-ordinates of the particle. (c is simply a normalisation constant.)

Observables are given by the square of the wave function $|\Psi|^2$.

If we consider a number of identical, indistinguishable particles, then clearly interchanging a pair of particles is quite unobservable.

$$|\Psi(2,1,3)|^2 = |\Psi(1,2,3)|^2$$

This has two possible solutions

$$\Psi(2,1,3) = +\Psi(1,2,3) \qquad (1)$$

$$\Psi(2,1,3) = -\Psi(1,2,3) \qquad (2)$$

These two cases have important physical consequences. E.g. if two particles are in identical quantum states

$$\Psi(1,2) = c\,\Psi_A(1)\,\Psi_A(2)$$

case (1) implies $c_1\,\Psi_A(2)\,\Psi_A(1) = c_1\,\Psi_A(1)\,\Psi_A(2)$
which is clearly satisfied by any c_1,

case (2) implies $c_2\,\Psi_A(2)\,\Psi_A(1) = -\,c_2\,\Psi_A(1)\,\Psi_A(2)$
which is only satisfied if $c_2 = 0$ – the wave function is zero.

Particles obeying the two conditions have completely different behaviours.

Bosons have wave functions which are symmetric under the interchange of identical particles. They obey Bose-Einstein statistics, showing constructive interference of identical single particle wave-functions.

$$\Psi = \tfrac{1}{\sqrt{2}}\left(\Psi_A(1)\Psi_B(2) + \Psi_A(2)\Psi_B(1)\right)$$

Fermions have wave functions which are antisymmetric under the interchange of identical particles. They obey Fermi-Dirac statistics, showing destructive interference of identical single particle wave functions. In particular, no two identical fermions can occupy wave functions with identical quantum numbers.

$$\Psi = \tfrac{1}{\sqrt{2}}\left(\Psi_A(1)\Psi_B(2) - \Psi_A(2)\Psi_B(1)\right)$$

Fermions are particles with "half integer" spin, i.e. $^1/_2\,\hbar$, $^3/_2\,\hbar$, $^5/_2\,\hbar$, … (e.g. proton, neutron, electron, neutrino, quarks, …).
They include the constituents particles of matter.

For each particle, there is a distinct antiparticle
e.g. $e^- \leftrightarrow e^+$
$\nu_e \leftrightarrow \bar{\nu}_e$ both neutral, but different.
They obey conservation laws – they are only produced as fermion-antifermion pairs.

Bosons have integer spin, i.e. 0, $\hbar$, $2\hbar$, … (e.g. photons, π (pi meson) and other mesons, $W^\pm$, Z, gluon …)
They include the quanta of fields, i.e. the carriers of forces.

They can be created and destroyed e.g. $e^- + e^- \rightarrow e^- + e^- + \gamma$
They are their own antiparticles e.g.
$\pi^+ \quad \pi^-$
$\pi^0 \leftrightarrow \pi^0$
$\pi^- \quad \pi^+$

Leptons and Quarks

The fundamental fermions are believed to be the electron-like particles known as leptons and the quarks.

(As we will see later, the proton and neutron – examples of baryons – are made of quarks. So are mesons, and together they make up the hadrons.)

The basic constituents are the electron and neutrino (leptons) and u and d quarks.
The leptons have an associated lepton number L which is (as far as we know) absolutely conserved.

e^- and ν have $L = +1$

e^+ and $\bar{\nu}$ have $L = -1$

In fact, though the above 4 particles are all that is required to make the present day universe, the pattern is repeated, occurring 3 times, with heavier, unstable versions of the ordinary particles.

So, in the lepton sector we have:

	e^-	μ^-	τ^-	
mass	0.511	105.7	1777	MeV/c^2
	ν_e	ν_μ	ν_τ	
very small	<2 eV/c^2	<2 eV/c^2	<2 eV/c^2	

and their antiparticles.

[As far as this course is concerned, we will treat all three species of neutrino as massless. Recent evidence indicates that, though these masses are certainly very small – probably less than 0.1 eV – they are actually not zero. This may have important consequences for our theories of the particle families. For more information on these recent results, please see the web pages – but note this is definitely non-examinable!]

Each generation has its own distinct lepton number, L_e, L_μ, L_τ. This is what makes the different neutrinos distinct, and forbids $\mu \rightarrow e\,\gamma$.

As we will see later, the quark types can be changed by the weak interaction, so there is only a global baryon number B.
Quarks have $B = +\frac{1}{3}$, antiquarks have $B = -\frac{1}{3}$.

e.g. proton (uud) has $B = +1$; neutron (udd) has $B = +1$;
$\bar{p}$ ($\bar{u}\bar{u}\bar{d}$) has $B = -1$; π^+ ($u\bar{d}$) has $B = 0$.

We will discuss the heavier quarks, and the allowed combinations of quarks which form hadrons, later.

The Standard Model of Particle Physics

Fermions: Leptons		Fermions: Quarks		Bosons
e	ν_e	d	u	γ
μ	ν_μ	s	c	$W^\pm$, Z^0
τ	ν_τ	b	t	g
				H^0

Reading for next week from "Ideas of Particle Physics":

- *Chapter 5 – Four forces*
- *Chapter 7 – Mesons, Yukawa etc.*

Go to Example C

What you should have learned this week

- **The effects of symmetry for fermions and bosons**
- **Examples of fermions & bosons; antiparticles**
- **Fundamental fermions**
  - **6 leptons, and lepton number**
  - **u & d quarks; baryons, mesons & hadrons**
- **4-vectors**
- **2-body decay and scattering**
- **The conserved invariant $(\Sigma E)^2 - (\Sigma \underline{p})^2$**