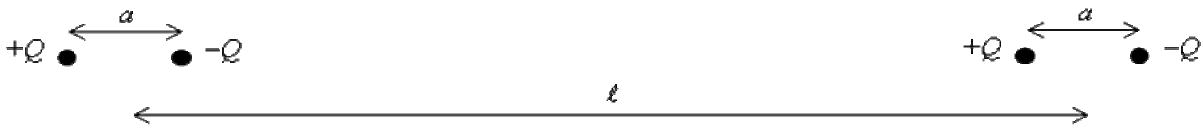
PHY102 Electricity
Dipole Calculation (from lecture)

Two dipoles, with charges $\pm Q$ and separation a, are placed parallel and a distance ℓ apart:

(a) parallel to the line of the dipoles
(b) perpendicular to the line of the dipoles.
Compare the force between the dipoles in the two cases.

a)



To calculate the total force, we must consider the effect of each charge in the left dipole on each charge in the right dipole:

$$F = \frac{2Q^2}{4\pi\varepsilon_0 \ell^2} - \frac{Q^2}{4\pi\varepsilon_0 (\ell - a)^2} - \frac{Q^2}{4\pi\varepsilon_0 (\ell + a)^2}$$

(The first term represents the force between the two like-sign pairs of charges; the second is that between the $-Q$ on the left and $+Q$ on the right; the third is that between $+Q$ on the left and the $-Q$ on the right.)

$$F = \frac{Q^2}{4\pi\varepsilon_0 \ell^2}\left(2 - \left(1 - \frac{a}{\ell}\right)^{-2} - \left(1 + \frac{a}{\ell}\right)^{-2}\right)$$

Using the binomial expansion $(1+y)^n = 1 + ny + \frac{n(n-1)}{2!}y^2 + \cdots$

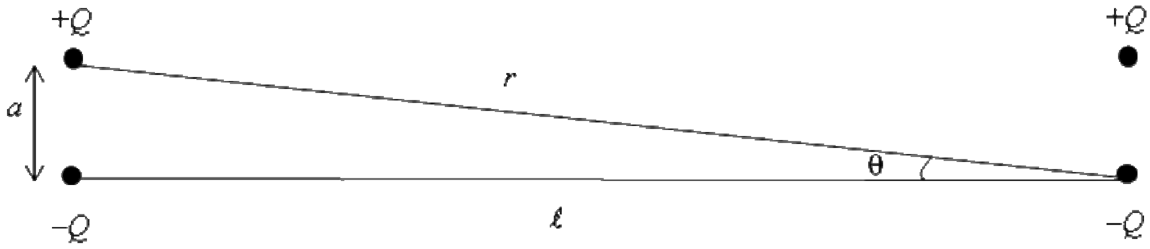
$$F \simeq \frac{Q^2}{4\pi\varepsilon_0 \ell^2}\left(2 - \left(1 + \frac{2a}{\ell} + \frac{2\cdot 3}{2}\left(\frac{a}{\ell}\right)^2 \cdots\right) - \left(1 - \frac{2a}{\ell} + \frac{2\cdot 3}{2}\left(\frac{a}{\ell}\right)^2 \cdots\right)\right)$$

$$= -\frac{Q^2}{4\pi\varepsilon_0 \ell^2}\frac{6a^2}{\ell^2} = -\frac{3Q^2a^2}{2\pi\varepsilon_0 \ell^4} = -\frac{3p^2}{2\pi\varepsilon_0 \ell^4} \quad \text{where } p = Qa$$

Notes:

1. The force between two dipoles is proportional to the reciprocal of their separation to the forth power (rather than squared for the force between two point charges).
2. It only depends on the product of Q and a (i.e. p), not on either of them individually.
3. For this configuration, the force is attractive (negative).

b)



Again, to calculate the total force, we must consider the effect of each charge in the left dipole on each charge in the right dipole. However, the forces between the unlike pairs of charges are not in the same direction, and the vertical components will cancel. We must therefore only consider the horizontal components. The unlike charges are a distance *r* apart, where $r^2 = \ell^2 + a^2$.

$$F = \frac{2Q^2}{4\pi\varepsilon_0 \ell^2} - \frac{2Q^2}{4\pi\varepsilon_0 \left(\ell^2 + a^2\right)} \cos\theta$$

(The first term represents the force between the two like-sign pairs of charges; the second is the horizontal component of the force between the two unlike pairs of charges.)

Now $\cos\theta = \frac{\ell}{r} = \frac{\ell}{\left(\ell^2 + a^2\right)^{1/2}}$.

$$F = \frac{2Q^2}{4\pi\varepsilon_0 \ell^2} - \frac{2Q^2 \ell}{4\pi\varepsilon_0 \left(\ell^2 + a^2\right)^{3/2}}$$ but $\left(\ell^2 + a^2\right)^{3/2} = \ell^3 \left(1 + \frac{a^2}{\ell^2}\right)^{3/2}$

$$F = \frac{2Q^2}{4\pi\varepsilon_0 \ell^2} \left(1 - \frac{\ell}{\ell \left(1 + \frac{a^2}{\ell^2}\right)^{3/2}} \right) = \frac{2Q^2}{4\pi\varepsilon_0 \ell^2} \left(1 - \left(1 + \frac{a^2}{\ell^2}\right)^{-3/2} \right)$$

$$= \frac{2Q^2}{4\pi\varepsilon_0 \ell^2} \left(1 - \left(1 - \frac{3}{2}\frac{a^2}{\ell^2} \cdots \right) \right) = \frac{3Q^2 a^2}{4\pi\varepsilon_0 \ell^4} = \frac{3p^2}{4\pi\varepsilon_0 \ell^4}$$

Notes:

1. Again, the force between the two dipoles is proportional to the reciprocal of their separation to the forth power.
2. In this case, the force is repulsive.
3. Its magnitude is half as big as in part a).