PHY102 Electricity
Topic 6 (Lectures 9 & 10) – Electric Current and Resistance

In this topic, we will cover:
1) Current in a conductor
2) Resistivity
3) Resistance
4) Ohm's Law
5) The Drude Model of conduction

Reading from Young & Freedman:
For this topic, read the introduction to chapter 25 and sections 25.1 to 25.3 & 25.6.

Introduction

The earlier part of this course has been concerned with electrostatics, that is electric charges at rest. We will now examine charges in motion. We will see that, in a conductor, electrons are always moving within the body of the material. However, this random motion does not result in the transport of charge from one point to another, and so does not constitute an electric current. When an electric field resulting from some potential difference acts on free charges such as electrons, however, there is a net motion superimposed on any random movement, and an electric current flows.

Electrons in Conductors

A conductor such as a metal consists of atoms arranged in a crystal lattice each of which has one or more electrons which are very weakly bound to the rest of the atom. They are therefore free to move around within the body of the conductor. We have already seen that if a steady electric field is applied, they will move in such a way as to build up a charge density which produces an electric field neutralising the applied external field. In fact, even in the absence of an electric field, the electrons are in constant motion. This is partly due to the thermal energy they possess as a consequence of their not being at absolute zero, and partly due to quantum mechanical effects that you will meet in your second year courses. As a result, electrons typically have velocities of the order of 10^6 m s^{-1}. They are also continually colliding with the stationary ions in the crystal lattice, resulting in scattering of the electrons into different directions. Since their velocities are essentially random there is no net flow of charge through the material, and no current flows.

When an electric field is applied to the conductor, the charged electrons experience a force $\mathbf{F} = q\,\mathbf{E}$. As we discussed previously, a completely free charge, such as an electron, would then be accelerated with a constant acceleration by this force. In a metal, however, the electrons are only accelerated for a short time before they collide with the ions in the lattice, and have their motion randomised again. The consequence is that the electrons have a small *drift velocity* superimposed on their random motion. Although this drift velocity is very much less than the random motion (perhaps 10^{-4} m s^{-1} compared with 10^6 m s^{-1}) because the drift velocity of all the electrons is in the same direction it constitutes a net transfer of charge in one direction, and this is an electric current.

In fact, electric conduction is usually caused by the movement of negative electrons. The force on them, and hence their acceleration and drift velocity, is in the opposite direction to the electric field. Electric currents were studied long before the constituents of atoms were understood, and the conventional direction for a current is that in which positive particles would move, i.e. from a positive charge or high potential towards a negative charge or lower potential. Thus the arrow indicating conventional current direction points in the opposite direction from that taken by the drifting electrons.

Electric Current and Circuits

Current is the rate at which charge flows. In a wire, this could be the rate of charge flow past a given point, or in bulk material it could be the flow through a given surface. If in a time Δt a charge ΔQ flows, then the average electric current is given by

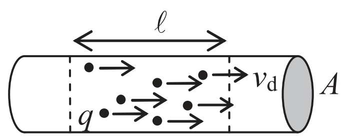
$$I_{\text{av}} = \frac{\Delta Q}{\Delta t},$$

and the instantaneous current is
$$I = \frac{\mathrm{d}Q}{\mathrm{d}t}. \qquad [1]$$

The S.I. unit of current is the Ampère or Amp (A). 1 A = 1 Coulomb per second.

For a current to flow along a wire, there must be a potential difference between its ends. This may be provided by connecting it to a battery, which raises positive charges from a low potential (at the negative terminal) to high potential (at the positive terminal). Note that current can only flow continuously through a closed loop or circuit, such as that formed by the wire and battery. As much charge enters one end of the wire as leaves the other end. The wire does not gain any net charge. (The wire acts somewhat like a hose pipe, where as much water enters one end as leaves the other.)

Current Density



The figure alongside shows particles of charge q moving with drift velocity v_d along a wire. (Since the random motion does not contribute to conduction, this is ignored.) If there are n charges per unit volume, the total charge within a cylinder of length ℓ and cross sectional area A is $\Delta Q = n(A\ell)q$. This charge will take a time $\Delta t = \ell/v_d$ to pass through the end of the cylinder. The current, $I = \Delta Q/\Delta t$ is therefore

$$I = nAqv_{\text{d}}.$$

The average current density J is just the current flowing per unit area at right angles to the current, so
$$J = \frac{I}{A}. \qquad [2]$$

Note that current is a scalar but current density is a vector with direction given by the local direction of the drift velocity. We can therefore write

$$\mathbf{J} = nq\mathbf{v}_{\text{d}}. \qquad [3]$$

If the carriers have negative charge, then $\mathbf{J}$ is opposite in direction to $\mathbf{v}_d$.

Resistance and Resistivity

The current I flowing through a component such as a wire depends on the applied potential difference V between the terminals of the component. We define the *resistance* R of the component as
$$\boxed{R = \frac{V}{I}}. \qquad [4]$$

The S.I. unit of resistance is the ohm (Ω), defined as one volt per amp.

A less frequently used parameter is the conductance G which is the reciprocal of resistance, and its units are written as mho (= Ω^{-1})

$$G = \frac{1}{R} = \frac{I}{V}.$$

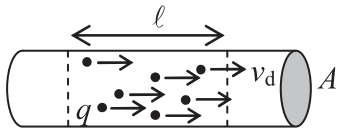
As we will see, the resistance, which relates current to voltage, depends on the geometry of the material through which the current flows. (We have already see how it depends on the cross sectional area.) We can instead consider the relationship between current density and electric field.

Equation [3] showed that current density is proportional to drift velocity, and for a given material at constant temperature, the drift velocity is normally proportional to the electric field. The relationship between **J** and **E** can be expressed as

$$\boxed{\mathbf{J} = \sigma\mathbf{E} = \frac{1}{\rho}\mathbf{E}}. \qquad [5]$$

The constant ρ is known as the *resistivity* of the medium, with S.I. units ohm metre (Ω m), and $\sigma = 1/\rho$ is the conductivity. The resistivity is a property of the material and does not depend on its shape.

Consider the section of wire examined previously. If the potential difference across the length ℓ is V, then $E = V/\ell$. Substituting this and $J = I/A$ into [5], we arrive at



$$\frac{I}{A} = \frac{1}{\rho}\frac{V}{\ell}$$

so

$$\boxed{R = \frac{V}{I} = \frac{\rho\ell}{A}}. \qquad [6]$$

In other words, the resistance of a sample is directly proportional to its length, and inversely proportional to its area.

Temperature Coefficient of Resistivity

The resistivity of a metal is due to collisions of the conduction electrons with ions in the lattice. As the temperature increases, the amplitude of vibration of the ions increases, and so does the probability of scattering the electrons. The resistivity of the material thus increases with temperature. Over modest temperature rises (of the order of 100°C or so), the change in resistivity is approximately proportional to the temperature difference, and this change can be written as

$$\rho(T) \simeq \rho_0\left(1 + \alpha(T - T_0)\right). \qquad [7]$$

Here $\rho(T)$ is the resistivity at temperature T and ρ_0 is the resistivity at some standard temperature T_0, often taken as 0°C or 20°C. α is known as the *temperature coefficient of resistivity*, and by rearranging [7] we can see this is just

$$\alpha = \frac{\rho - \rho_0}{\rho_0 \Delta T}, \qquad [8]$$

i.e. the fractional change in resistivity per degree. Its units are $°C^{-1}$. Since the resistance of a sample is proportional to resistivity, we also have the result

$$\boxed{R(T) \simeq R_0\left(1 + \alpha(T - T_0)\right)}. \qquad [9]$$

Electrical Power

When a current flows as a result of an applied potential difference, the charge carriers give up potential energy. They do not, however, continue to build up their kinetic energy, as the scattering from the lattice means that they only acquire a small drift velocity. Instead, the energy (supplied by the battery) is transferred to the lattice and normally appears as heat.

The power P dissipated by the electric current is equal to the rate at which charges lose potential energy. If a charge dQ passes through a circuit in time dt, this is $V\mathrm{d}Q/\mathrm{d}t$. Using [1] this can be written

$$\boxed{P = IV}. \qquad [10]$$

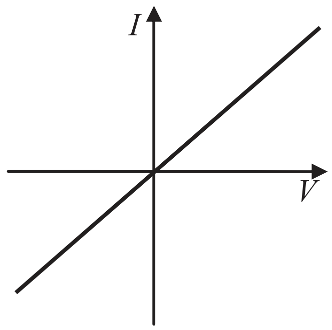
Equation [4] allows this to be rewritten in a number of useful forms:

$$\boxed{P = IV = I^2 R = \frac{V^2}{R}}. \qquad [11]$$

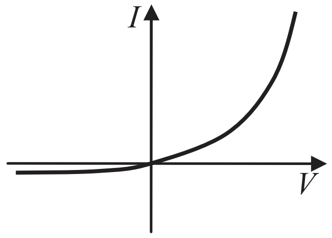
Ohm's Law

Ohm's Law states that, for certain conductors under appropriate conditions, the current flowing through the conductor is directly proportional to the potential difference across it. This can be written as $\frac{V}{I} = \text{constant}$.

We have already seen, in equation [4], that $V/I = R$, so Ohm's Law is really a statement that the resistance of many materials is not dependent on voltage or current. You should be aware, however, that many materials do not obey Ohm's Law. In a light bulb, for example, the filament becomes very hot as a result of the electrical power dissipated in it. This raises its resistance (according to equation [9]), so voltage certainly does not remain proportional to current. The figures alongside show in (a) a linear *I-V* relationship for an Ohmic device, such as a copper wire which remains at room temperature, and in (b) the non-linear *I-V* relationship for a non-Ohmic device – in this case a semiconductor diode which allows conduction in one direction much more easily than in the other.



(a) Ohmic conductor



(b) Non-Ohmic conductor

The Drude Model of Conduction

A classical (non-quantum mechanical) model of electrical conduction was proposed by Paul Drude in 1900. As we have already discussed, electrons are moving randomly due to their thermal energy. When an electric field $\mathbf{E}$ is applied, the electrons experience a force $-e\mathbf{E}$, so an acceleration $\mathbf{a} = -e\mathbf{E}/m$. Because of collisions with the ionic lattice, the velocity does not continue to increase. If the mean time between collisions is τ, at which point the electron's velocity is completely randomised, the electron achieves a drift velocity of

$$\mathbf{v}_\mathrm{d} = -\frac{e\mathbf{E}}{m}\tau . \qquad [12]$$

The constant τ is a property of the conducting material, but does not depend on the drift velocity (since this is so much smaller than the random thermal velocity). Using [3], $\mathbf{J} = nq\mathbf{v}_\mathrm{d}$, we have

$$J = \frac{ne^2\tau}{m} E = \frac{1}{\rho} E . \qquad [13]$$

so

$$\boxed{\rho = \frac{m}{ne^2\tau}}. \qquad [14]$$

Putting What You Have Learnt Into Practice

Question 6.1

Copper has a resistivity of 1.7×10^{-8} Ω m. A wire of diameter 1.5 mm and length 25 m is connected across a potential difference of 50 V. Calculate:

(a) the resistance of the wire,

(b) the current,

(c) the power dissipated in the wire.

Solution

(a) The cross sectional area is $A = \pi r^2$. Therefore

$$R = \frac{\rho l}{A} = \frac{1.7\times10^{-8}\times25}{\pi\times\left(0.75\times10^{-3}\right)^2} = 0.24\,\Omega .$$

(b) The current is $$I = \frac{V}{R} = \frac{50}{0.24} = 207\text{ A} .$$

(c) The power is $$P = \frac{V^2}{R} = \frac{50^2}{0.24} = 10.4\,\text{kW}$$

Question 6.2

A platinum resistance thermometer measures temperature by the change in the electrical resistance of a platinum wire. The coefficient of resistivity for platinum is 3.92×10^{-3} °C^{-1}. At a temperature of 20.0°C, the thermometer has a resistance of 50.0 Ω; when immersed in a crucible containing melting indium its resistance is 76.8 Ω. What is the melting point of indium?

Solution

The expression for resistance as a function of temperature is

$$R(T) \simeq R_0\left(1+\alpha\left(T-T_0\right)\right).$$

Hence $$T - T_0 = \frac{R - R_0}{\alpha R_0} = \frac{76.8-50.0}{3.92\times10^{-3}\times50.0} = 137\,°\text{C}$$

The melting point of indium is 20+137 = 157°C.

Question 6.3

An electric fire has a heating element rated at 1 kW when operating at 230 V.

(a) What is its resistance?

(b) What will be the power dissipation if the mains voltage drops to 210 V, assuming that the element obeys Ohm's Law?

Solution

(a) $$P = \frac{V^2}{R} \quad\Rightarrow\quad R = \frac{V^2}{P} = \frac{230^2}{1000} = 52.9\ \Omega .$$

(b) As we are given that the element obeys Ohm's Law, its resistance remains constant. Since $P = \frac{V^2}{R}$, power is proportional to the square of the voltage. The power at the reduced voltage is therefore given by

$$P_{210} = \left(\frac{210}{230}\right)^2 P_{230} = 0.834 \times 1000 = 834\ \mathrm{W}\ .$$

Question 6.4

Given that copper has a resistivity of $1.7\times10^{-8}\ \Omega$ m and has 8.5×10^{28} free electrons per cubic metre, calculate the mean time between collisions between a conduction electron and the ionic lattice according to the Drude model. If the copper is exposed to an electric field of 0.5 V m^{-1}, what average drift velocity will the electron achieve? (The mass of an electron is 9.11×10^{-31} kg.)

Solution

From the lecture notes, we saw $$\rho = \frac{m}{ne^2\tau}$$

Therefore $$\tau = \frac{m}{ne^2\rho} = \frac{9.11\times10^{-31}}{8.5\times10^{28}\times\left(1.6\times10^{-19}\right)^2\times1.7\times10^{-8}} = 2.5\times10^{-14}\ \mathrm{s}$$

Drift velocity $$v_\mathrm{d} = -\frac{eE}{m}\tau = -\frac{1.6\times10^{19}\times0.5}{9.11\times10^{-31}}\times2.5\times10^{-14} = -2.2\times10^{-3}\ \mathrm{m\,s^{-1}}\ .$$

(The negative sign simply means that the electron drifts in a direction opposite to that of the electric field.)

Problems from Young & Freedman for Topic 6:
Try to do exercises 25.1 to 25.27, 25.39 to 25.45, 25.54 to 25.60, 25.63 to 25.64 and 25.67. The later problems are more challenging. (Numerical answers to odd-numbered questions are available at the back of the book.)