PHY102 Electricity
Topic 1 (Lecture 1) – Electrostatics

In this topic, we will cover:
1) Electric charge
2) Conductors, insulators and induction
3) Coulomb's Law
4) The Principle of Superposition

Reading from Young & Freedman:
For this topic, read the introduction to chapter 21 and sections 21.1 to 21.3.

Introduction

Almost all the everyday phenomena that we experience are based on electromagnetism. This includes most properties of materials, including their bonding, friction between objects, chemical reactions, light and radio waves and the transmission of signals along nerve fibres. The only common non-electromagnetic force we experience is gravitation, which is only important between large objects, such as ourselves and the earth. (Other forces exist at the nuclear and sub-nuclear level, responsible for nuclear binding and radioactive decay.)

Electric charge is a fundamental property of matter which causes it to experience electrical (and magnetic) effects. Matter is composed of atoms, which consist of a tiny nucleus surrounded by electrons. The nucleus consists of protons and neutrons, and the protons carry a positive electric charge. The electrons carry an equal but opposite (negative) charge. Normal matter contains atoms with the same number of electrons as protons – they are therefore electrically neutral. An ion is an atom (or molecule) which has lost or gained one or more electrons, and so carries an overall electric charge.

The first systematic observations of electrical effects were of what was known as *frictional electricity*. When a plastic rod is rubbed with fur, some electrons are transferred from the fur to the plastic, so the latter gains a net negative charge. Similarly, if a glass rod is rubbed with silk, electrons are transferred from glass to silk, so the glass develops a net positive charge. (In fact, the word "electric" is derived from the Greek for amber, which was the original material observed to obtain a charge after rubbing.)

The study of electrical properties of charges at rest is known as *electrostatics*. Moving electric charges constitute an electric current, and as we shall see later in the course, electric currents generate magnetic effects. When both electric and magnetic effects are present, the interactions involved are known as *electromagnetic*. Although for much of the course we will treat electric and magnetic effects separately, a complete description of these effects requires a theory of electromagnetism.

Electric Charge

The S.I. unit of charge is the Coulomb, C. It is defined in terms of electric current, which is the rate of flow of charge. (This is because practically it is easier to make precise measurements of electric currents, through their magnetic effects.) The Coulomb is a very large unit. The charge on a proton is given by

$$e = 1.602\times10^{-19}\ \text{C}$$

and the charge on an electron is minus this amount. Electric charge appears only in these discrete amounts (it is said to be *quantised*). When an object is rubbed, the accumulated charge is a small fraction of a Coulomb – typically 10^{-8} C.

The normal symbol for charge is Q or q. Thus the electron and proton charges may be denoted

$$q_e = -e \qquad\qquad q_p = +e$$

Charge cannot be either created or destroyed. This fact is known as the conservation of charge:

In any isolated system, the total charge remains constant.

Conductors and Insulators

Materials such as glass and plastic maintain any static frictional electric charge. Charges at one point on their surfaces are not free to move to another part of the object. They are known as *insulators*. Other examples of insulators are dry wood and rubber. Other materials, such as metals and ionic solutions, allow charge to flow freely through them. They are known as *conductors*. Very pure, distilled water is a poor conductor, but normal tap water is quite a good conductor due to dissolved impurities which produce ions. Earth is a good conductor because of the water it contains. (A third group of materials is the semiconductors, such as silicon, germanium and gallium arsenide. These are intermediate in behaviour, showing some insulating and conducting properties, largely depending on their purity.)

Electrical conduction is the movement of electric charges. In a metal, some of the outer electrons are not strongly bound to an individual atom, and readily become free. They are then able to move in a sea of free charge which is only attached to the sample of metal as a whole. If extra electrons are added at one point, the forces they exert on the original electrons cause them to spread out, so that the extra charge is distributed through the conductor. In ionic solutions, it is the movement of the ions which is responsible for the electrical conduction.

Forces between Charges and Induction

Like charges (that is, two positive or two negative charges) repel each other. Similarly, two unlike charges (one positive and one negative) attract each other. We can exploit this fact in the process of *induction* to use an existing charge on one body to produce charge on other bodies. Consider what happens if we bring a positively charged glass rod close to a conducting body mounted on an insulating stand, as shown in figure 1(a). The positive charge on the rod will attract free electrons to the near side of the body leaving a deficit on the far side. (This is equivalent to positive charge being repelled to the far side.) Next, if we temporarily connect the far side of the body to ground with a conducting wire as in fig. 1(b), the excess positive charge will be neutralised by electrons being attracted from the ground. (Effectively, the positive charge leaks to ground.) This leaves the body with a net negative charge. Finally (figure 1(c)) if the wire and then the glass rod are removed, we are left with a negatively charged body.

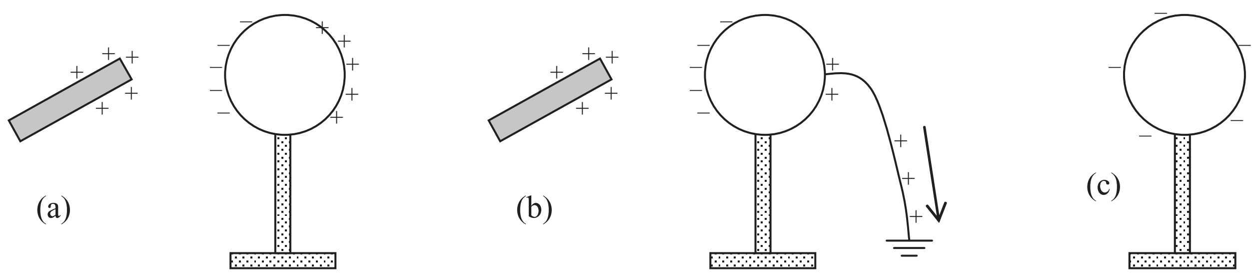


Figure 1 Charging by Induction

Think! How was charge conserved in the above process?

Coulomb's Law

The magnitude of the force between two charges depends on the size of the charges. It also depends on the distance between the charges. In 1784 Charles Augustin de Coulomb determined the force law for electrostatic charges, and this is known as Coulomb's Law.

The magnitude of the force between two point charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them. The direction of the force is along the line joining the charges.

(Note that Coulomb's law applies to *point* charges. If the charge is distributed in space, firstly the separation distance is not well defined, and secondly one charge may affect the distribution of the other charge. Also the charges must be at rest; moving charges will also experience magnetic forces.)

Consider a point particle carrying a charge q at a distance r from another particle carrying charge Q. Coulomb's law tells us that the magnitude of the force, F is

$$F = k\frac{qQ}{r^2} \qquad [1]$$

where k is a constant. In S.I. units, the value of k is

$$k = 8.99 \times 10^9 \text{ N m}^2 \text{ C}^{-2} .$$

The constant k is usually written as

$$k = \frac{1}{4\pi\varepsilon_0} . \qquad [2]$$

(We will see later why the factor of 4π is useful.) The quantity ε_0 is called the permittivity of free space, and has the value (as you could work out!)

$$\varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2} .$$

(We will see a more convenient way of expressing the units later in the course.)

Expressing Coulomb's law in vector form, we arrive at

$$\mathbf{F} = \frac{qQ}{4\pi\varepsilon_0 r^2}\hat{\mathbf{r}} \qquad [3]$$

where $\hat{\mathbf{r}}$ is a unit vector which has its origin at the "source of the force". For example, to find the force on q due to Q, the origin of $\hat{\mathbf{r}}$ must be placed at Q, as shown in figure 2. If the magnitude of F is positive (which it will be if both charges have the same sign), we have a repulsive force, tending to move the charges apart.

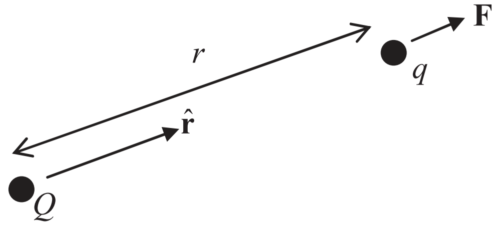


Figure 2 The force on q due to Q

The Principle of Superposition

The force exerted by one charge on another is not affected by any additional charges which may be present. This leads to the principle of linear superposition, which states that the force experienced by one charge as a result of a system of charges is just equal to the sum of the forces which would be produced by each of the other charges if it was present alone. Consider a system of charges q_1, q_2, q_3, … q_N. Using the notation $\mathbf{F}_{AB}$ = Force on A due to B, the net force $\mathbf{F}_1$ acting on q_1 is simply the vector sum

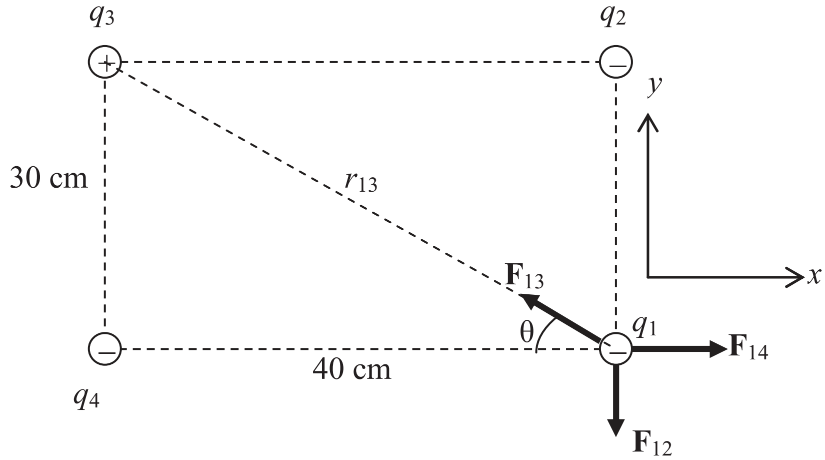
$$\mathbf{F}_1 = \mathbf{F}_{12} + \mathbf{F}_{13} + \cdots + \mathbf{F}_{1N}$$

Note on unit vectors: A unit vector is a dimensionless quantity that serves only to specify a direction in space. To simplify the manipulation of vectors, it can be convenient to introduce the unit vectors **i**, **j**, **k**, that lie along the *x*, *y* and *z* axes of a given coordinate system respectively. Since unit vectors have unit length, $|\mathbf{i}| = |\mathbf{j}| = |\mathbf{k}| = 1$. In general, any vector may be written as the sum of three vectors parallel to each of the coordinate axes:

$$\mathbf{V} = V_x\mathbf{i} + V_y\mathbf{j} + V_z\mathbf{k}$$

Putting What You Have Learnt Into Practice

Question 1.1
Four point charges are arranged at the corners of a rectangle as shown below. If $q_1 = -3$ μC, $q_2 = -5$ μC, $q_3 = 13$ μC and $q_4 = -15$ μC, find the net force on q_1 due to the other three charges.



Solution
The directions of the forces on q_1 and the coordinate axes are shown in the figure. The magnitude of the force on q_1 due to q_2 is:

$$F_{12} = \frac{|q_1 q_2|}{4\pi\varepsilon_0 r_{12}^2} = \frac{(3\times10^{-6})\times(5\times10^{-6})}{4\pi\times8.85\times10^{-12}\times0.3^2} = 1.5 \text{ N}$$

Similarly $F_{14} = \frac{|q_1 q_4|}{4\pi\varepsilon_0 r_{14}^2} = \frac{(3\times10^{-6})\times(15\times10^{-6})}{4\pi\times8.85\times10^{-12}\times0.4^2} = 2.5 \text{ N}$

$$F_{13} = \frac{|q_1 q_3|}{4\pi\varepsilon_0 r_{13}^2} = \frac{(3\times10^{-6})\times(13\times10^{-6})}{4\pi\times8.85\times10^{-12}\times0.5^2} = 1.4 \text{ N}$$ (note that $r_{13} = \sqrt{r_{12}^2 + r_{14}^2} = \sqrt{0.3^2 + 0.4^2} = 0.5$ m

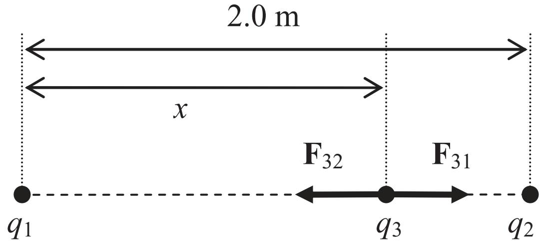
The overall force on q_1 is given by

$$F_{1x} = F_{14} - F_{13}\cos\theta = 2.5 - 1.4\times0.8 = 1.4 \text{ N}$$

$$F_{1y} = -F_{12} + F_{13}\sin\theta = -1.5 + 1.4\times0.6 = -0.7 \text{ N}$$

Or $$\boxed{\mathbf{F}_1 = 1.4\mathbf{i} - 0.7\mathbf{j} \text{ N}}$$

Question 1.2
Three positive point charges lie along a line as shown in the figure below. The charges q_1 and q_2 are separated by 2.0 m and are 15 μC and 6 μC respectively, while q_3 is between them. If there is no net force on q_3, how far is it situated from q_1?



Solution
Since we are not told whether q_3 is positive or negative, we do not know whether the forces between it and the other two charges are repulsive or attractive, but we do know they are both the same. By Coulomb's law, the magnitudes of the forces are

$$F_{31} = \frac{|q_3 q_1|}{4\pi\varepsilon_0 r_{31}^2} = \frac{|q_3 q_1|}{4\pi\varepsilon_0 x^2} \qquad F_{32} = \frac{|q_3 q_2|}{4\pi\varepsilon_0 r_{32}^2} = \frac{|q_3 q_2|}{4\pi\varepsilon_0 (2-x)^2}$$

Setting these two equal, and cancelling common terms, we have

$$\frac{q_1}{x^2} = \frac{q_2}{(2-x)^2}$$

Cross-multiplying and taking the square root of each side then gives us

$$(2-x)\sqrt{q_1} = x\sqrt{q_2}$$

$$2\sqrt{q_1} = x\sqrt{q_2} + x\sqrt{q_1} = x\left(\sqrt{q_1} + \sqrt{q_2}\right)$$

$$x = \frac{2\sqrt{q_1}}{\sqrt{q_1} + \sqrt{q_2}} = \frac{2\sqrt{15\times10^{-6}}}{\sqrt{15\times10^{-6}} + \sqrt{6\times10^{-6}}} = 2\frac{\sqrt{5}}{\sqrt{5} + \sqrt{2}} = 1.225 \text{ m}$$

There is no net force when q_3 is 1.225 m from q_1.

Question 1.3

The electron and proton in a hydrogen atom are 0.53×10^{-10} m apart. What is the ratio of the gravitational to the electrostatic force between them?

Solution

The magnitude of the electrostatic force is given by $F_{\text{ES}} = \dfrac{e^2}{4\pi\varepsilon_0 r^2}$.

The magnitude of the gravitational force is given by $F_{\text{G}} = \dfrac{Gm_{\text{e}}m_{\text{p}}}{r^2}$.

The ratio of the forces is thus

$$\frac{F_{\text{G}}}{F_{\text{ES}}} = \frac{4\pi\varepsilon_0 Gm_{\text{e}}m_{\text{p}}}{e^2} = \frac{4\pi\times8.85\times10^{-12}\times6.67\times10^{-11}\times9.11\times10^{-31}\times1.67\times10^{-27}}{\left(1.6\times10^{-19}\right)^2} = 4.4\times10^{-40}$$

Note firstly that this is extremely small! This explains why when we deal with interactions between elementary particles, gravitation can safely be ignored. It also explains why a charged comb can lift a piece of paper and so overcome the gravitational attraction of the whole earth!

Secondly, because Newton's law of gravitation and Coulomb's law are both inverse square laws they have the same dependence on separation. This means that the actual distance *r* disappears from the above calculation.

Problems from Young & Freedman for Topic 1:
Try to do exercises 21.1 to 21.24 and 21.63 to 21.85. The later problems are more challenging. (Numerical answers to odd-numbered questions are available at the back of the book.)