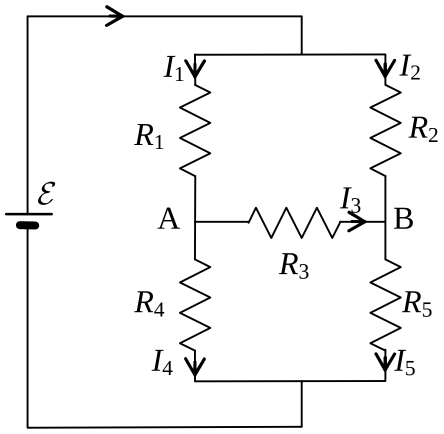
<u>Example of Use of Kirchhoff's Rules</u>



What is the effective resistance of the network above? When connected to a source of EMF as shown, what are the potentials at points A & B (with respect to the negative side of the battery)?

Use the following component values: $\mathcal{E}$ = 3 V, R_1 = 3 Ω, R_2 = 4 Ω, R_3 = 10 Ω, R_4 = 7 Ω, R_5 = 4 Ω.

Applying Kirchhoff's junction rule at A: $I_1 - I_3 - I_4 = 0$ (1)

Applying Kirchhoff's junction rule at B: $I_2 + I_3 - I_5 = 0$ (2)

Applying Kirchhoff's loop rule: clockwise, upper loop:

$$-I_2R_2 + I_3R_3 + I_1R_1 = 0 \quad (3)$$

Applying Kirchhoff's loop rule: clockwise, lower loop:

$$-I_5R_5 + I_4R_4 - I_3R_3 = 0 \quad (4)$$

Applying Kirchhoff's loop rule: clockwise, left loop:

$$\mathcal{E} - I_1R_1 - I_4R_4 = 0 \quad (5)$$

(Note: we have 5 unknowns – the 5 currents – so require 5 independent equations.)

(1) $\Rightarrow$ $I_3 = I_1 - I_4$ (1')

(2) $\Rightarrow$ $I_3 = I_5 - I_2$

$\Rightarrow$ $I_5 - I_2 = I_1 - I_4$ (2')

(3) $\Rightarrow$ $-I_2R_2 + (I_1 - I_4)R_3 + I_1R_1 = 0$ (6)

(4) $\Rightarrow$ $-I_5R_5 + I_4R_4 - (I_1 - I_4)R_3 = 0$ (7)

(6)×R_5 – (7)×R_2 $\Rightarrow$ $-I_2R_2R_5 + (I_1 - I_4)R_3R_5 + I_1R_1R_5 - \left(-I_5R_2R_5 + I_4R_2R_4 - (I_1 - I_4)R_2R_3\right) = 0$

$\Rightarrow$ $(I_5 - I_2)R_2R_5 + (I_1 - I_4)R_3R_5 + I_1R_1R_5 - I_4R_2R_4 + (I_1 - I_4)R_2R_3 = 0$

Or, using (2') $(I_1 - I_4)(R_2R_5 + R_3R_5 + R_2R_3) + I_1R_1R_5 - I_4R_2R_4 = 0$

It is probably easier at this point to substitute in the component values:

$$(4\times4 + 10\times4 + 4\times10)(I_1 - I_4) + 3\times4I_1 - 4\times7I_4 = 0$$

$$108I_1 - 124I_4 = 0 \qquad \Rightarrow \quad I_4 = \frac{108}{124} I_1 = \frac{27}{31} I_1$$

(5) $\Rightarrow$ $\mathcal{E} - I_1 R_1 - I_4 R_4 = 0 \qquad \Rightarrow \quad 3 - 3I_1 - 7 \times \frac{27}{31} I_1 = 0$

$\Rightarrow$ $I_1 = 0.330\ \text{A} \qquad I_4 = \frac{27}{31} I_1 = 0.287\ \text{A}$

(1) $\Rightarrow$ $I_3 = I_1 - I_4 = 0.043\ \text{A}$

(3) $\Rightarrow$ $-4I_2 + 10I_3 + 3I_1 = 0 \qquad \Rightarrow \quad I_2 = \frac{10I_3 + 3I_1}{4} = 0.355\ \text{A}$

(2) $\Rightarrow$ $I_5 = I_2 + I_3 = 0.398\ \text{A}$

$$I_{\text{Total}} = I_1 + I_2 = 0.685\ \text{A}$$

Effective resistance $R_{\text{Eff}} = \frac{\mathcal{E}}{I_{\text{Total}}} = \frac{3}{0.685} = 4.38\,\Omega$

$$V_{\text{A}} = I_4 R_4 = 0.287 \times 7 = 2.009\ \text{V}$$

$$V_{\text{B}} = I_5 R_5 = 0.398 \times 4 = 1.592\ \text{V}$$